Einstein, relativity and the big bang

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Overview

100 years of relativity

*The special theory of relativity (1905)*
*The general theory of relativity (1915)*

Relativity and the universe

*The static models of Einstein and de Sitter*
*The dynamic models of Friedman and Lemaître*

The expanding universe (1930)

*The recession of the galaxies*
*Einstein’s ‘big bang’ models of 1931 and 1932*
*New research into Einstein’s models of 1931 and 1932*
*Einstein’s steady-state model*

Conclusions: Einstein and the big bang
The special theory of relativity (1905)

Two new principles for inertial observers

- Invariance of laws of physics (including electromagnetism)
- Invariance of the speed of light

Implications for space and time

- Space, time not absolute: distorted by motion

Predictions

- Length contraction; time dilation
- Mass increase; equivalence of mass and energy

Space-time

- Space + time = space-time
- 4-dimensional entity

\[ ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \]
Evidence for special relativity

- **Mass increase**
  
  *The experiments of Kaufmann and Bucherer*

- **Time dilation**
  
  *The long-lived muon*
  
  $2 \mu s \rightarrow 22 \mu s$

- **Invariance of the speed of light**
  
  *Many experiments to measure c*

- **Particle experiments at the LHC**
  
  *Maximum velocity = c*
  
  *Mass increase*
  
  *Particle creation*
The general theory of relativity (1915)

Relativity and accelerated frames?
Relativity and gravity?

Two new principles

Principle of equivalence
Mach’s principle

Predictions

Space-time distorted by mass
Gravity = curvature of space-time

G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}

Empirical evidence

Orbit of Mercury: bending of starlight (Eddington, 1919)
Evidence for general relativity

- **Bending of distant light by stars**
  *Gravitational lensing*

- **Gravitational redshift**
  *Shift in wavelength of light due to grav. field*

- **Gravitational time dilation**
  *GPS corrections*

- **Black holes**
  *Centre of galaxies*

- **Gravitational waves**
  *Hulse–Taylor binary system*
II Relativity and the universe

- **Apply general relativity to the cosmos (1917)**
  
  *Ultimate test for new theory of gravitation*

- **Assumptions**
  
  - Static universe
  - Isotropic and homogeneous
  - Null solution

  
  \[ G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \]

- **Cosmological constant \( \lambda \)**
  
  - Add term to GFE for non-zero solution
  - Universe of closed curvature: no boundary problem
  - Cosmic radius and matter density defined by \( \lambda \)

  \[ G_{\mu\nu} + \lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \]

  \[ \lambda = \frac{k \rho}{2} = \frac{1}{R^2} \]
The de Sitter universe (1917)

- Apply general relativity to the cosmos
  Include cosmological constant

- ‘Empty’ universe solution
  Reasonable approximation
  Curvature of space proportional to cosmic constant

- Disliked by Einstein
  Conflict with Mach’s principle
  Singularity problem?  
  Beginning of Einstein’s dislike for cosmic constant

- Interest from astronomers
  Radiation from matter redshifted – Slipher effect?
  Static or non-static model? (Weyl 1923, Lemaître 1925)
Friedman models of the cosmos

**Time-varying solutions (1922)**

*Universe of time-varying radius*

*Assume positive spatial curvature*

*Two independent differential equations from GFE*

\[ \frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho, \]

\[ \frac{R'^2}{R^2} + \frac{2RR''}{R^2} + \frac{c^2}{R^2} - \lambda = 0. \]

**Expanding sphere**

*Density of matter decreases over time*

**Ignored by community**

*Considered ‘suspicious’ by Einstein*

*Mathematical correction, later retracted*

“To this a physical reality can hardly be ascribed”

**Negative spatial curvature (1924)**

*Cosmic evolution, geometry depends on matter content*
Lemaître’s universe (1927)

- **Expanding model of the cosmos from GR**
  - Similar to Friedman 1922 model
  - Starts from static Einstein universe

- **Redshifts of nebulae = expansion of space?**
  - Redshifts from Slipher, distances from Hubble
  - \( H = 585 \text{ km s}^{-1} \text{ Mpc}^{-1} \)

- **Ignored by community**
  - Belgian journal (in French)
  - Rejected by Einstein: “Votre physique est abominable”
  - Lemaître informed of Friedman’s solution
  - Einstein not up-to-date with astronomy?
III Astronomy and the universe

The ‘Great Debate’ (1900-1920)

Spiral nebulae = clusters of stars?

Galaxies beyond Milky Way?

Light from many spirals red-shifted (Slipher 1915, 1917)

The Hooker telescope (1917)

100-inch reflector

Edwin Hubble (1921)

Distance of 2 spirals

Cepheid variables resolved in nebulae

Leavitt’s period-luminosity relation

Spirals far beyond Milky Way (1925)

A universe of galaxies
Motion of nebulae: redshift

Frequency of light depends on motion of source relative to observer

Doppler Effect

Measure motion of nebulae by measuring light emitted

Light from most nebulae red shifted

Vesto Slipher
Hubble’s law

- **A redshift/distance relation for the nebulae?**
  *Motivation: establishing distances of all nebulae*

- **Combined 24 distances with redshifts**
  *Redshifts from Slipher: not acknowledged*

- **Linear relation (Hubble, 1929)**
  \[ H = 500 \text{ km s}^{-1} \text{ Mpc}^{-1} : \text{some errors} \]
  *Most important data point not shown*

- **Landmark result in astronomy**
  *Not cosmology*
The expanding universe

- **RAS meeting (1930)**
  
  *Eddington, de Sitter*
  
  If redshifts are velocities, and if effect is non-local
  
  Static cosmic models don’t match observations

- **Time-varying universe?**
  
  *Hubble’s law = expansion of space?*

- **Lemaître expanding model**
  
  *Eddington contacted by Lemaître*
  
  1927 model republished in English (1931)

- **Friedman-Lemaître models circulated**
  
  *Time-varying radius*
  
  *Time-varying density of matter*
  
  *Evolving universe*
The expanding, evolving universe (1930-)

- **Eddington (1930, 31)**
  *On the instability of the Einstein universe*
  Expansion caused by condensation?

- **Tolman (1930, 31)**
  *On the behaviour of non-static models*
  Expansion caused by annihilation of matter?

- **de Sitter (1930, 31)**
  *Further remarks on the expanding universe*
  Expanding universes of every flavour

- **Einstein (1931, 32)**
  *Friedman-Einstein model*  \( k = 1, \lambda = 0 \)
  *Einstein-de Sitter model*  \( k = 0, \lambda = 0 \)

*Occam’s razor?*
Einstein's 1931 model

**Einstein’s first dynamic model of the cosmos**

*Often cited, rarely read (not translated)*

Adopts Friedman 1922 model

*Time-varying, closed universe: $k = 1$*

**Cosmic constant redundant**: $\lambda g_{\mu\nu} = 0$

**Extraction of parameters!**

*Radius, density of matter*

$R \sim 10^8$ lyr, $\rho \sim 10^{-26}$ g/cm$^3$

**Timespan problem**

$10^{10}$ yr: conflict with astrophysics

*Attributed to simplifying assumptions (homogeneity)*
Einstein’s 1931 model revisited

First translation into English
O’Raifeartaigh and McCann 2014

Anomalies in calculations of radius and density
\[ P \sim 10^8 \text{ lyr}, \ \rho \sim 10^{-26} \text{ g/cm}^3 \]
*Should be* \( P \sim 10^9 \text{ lyr}, \ \rho \sim 10^{-28} \text{ g/cm}^3 \)

Source of error?
*Oxford*: \( D^2 \sim 10^{-53} \text{ cm}^2 \) (should be \( 10^{-55} \text{ cm}^2 \))
Time miscalculation \( t \sim 10^{10} \text{ yr} \) (should be \( 10^9 \text{ yr} \))
Non-trivial error: misses conflict with radioactivity

Not a cyclic model
“Model fails at \( P = 0 \)”
Contrary to what is often stated
Curvature not a given in dynamic models
Not observed empirically
Remove spatial curvature (Occam’s razor)

\[ ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2dt^2 \]

Simplest Friedman model
Time-varying universe with \( \lambda = 0, k = 0 \)
Important hypothetical case: critical universe
Critical density: \( \rho = 10^{-28} \text{ g/cm}^3 \)

Becomes standard model
Despite high density of matter
Despite age problem

Time evolution not considered in paper – see title
Einstein-de Sitter model revisited

Einstein’s cosmology review of 1933
Review of dynamic models from first principles
Culminates in Einstein-de Sitter model
Cosmic constant banished
Possibility of flat geometry

Parameters extracted
Critical density of $10^{-28}$ g/cm$^3$: reasonable
Timespan of $10^{10}$ years: conflict with astrophysics
Attributed to simplifications (incorrect estimate)

Published in 1933!
French book; small print run
Intended for scientific journal; not submitted
Significant paper
Bonus: Einstein’s steady-state model

- **Unpublished manuscript**
  
  Archived as draft of F-E model (1931)
  
  Similar title, opening to F-E model

- **Something different**
  
  Cosmological constant
  
  “Constant matter density determines expansion”

- **Steady-state model**
  
  Continuous formation of matter from vacuum
  
  Fatal flaw; null solution
  
  Abandoned, not amended

Anticipates controversial theory (Hoyle)
New solution

“In what follows, I wish to draw attention to a solution to equation (1) that can account for Hubbel’s facts, and in which the density is constant over time”

Matter creation

“If one considers a physically bounded volume, particles of matter will be continually leaving it. For the density to remain constant, new particles of matter must be continually formed within that volume from space “

Dark energy

“The conservation law is preserved in that, by setting the λ-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”
Abandoned model

- **de Sitter line element**
  - Correct geometry

- **Simultaneous equations**
  - \( \alpha^2 = \frac{k c^2}{3 \rho} \)

- **Error in derivation**
  - **Null solution**

- **Einstein’s crossroads**
  - Realised problem on revision
  - Declined to amend GFE

- **Evolving models**
  - Less contrived and set \( \lambda = 0 \)
Taking $T_{44} = \rho c^2$ (all other components zero) in the time component of equation (1) we obtain
\[ (R_{44} - \frac{1}{2} g_{44} R) - \lambda g_{44} = \kappa \rho c^2. \]
This gives on analysis $\frac{3\alpha^2}{4} + \frac{3\alpha^2}{2} - \lambda c^2 = \kappa \rho c^2$
the second of Einstein’s simultaneous equations.

From the spatial component of equation (1), we obtain
\[ (R_{ii} - \frac{1}{2} g_{ii} R) - \lambda g_{ii} = 0. \]
This gives on analysis $\frac{3\alpha^2}{4} - \frac{3\alpha^2}{2} + \lambda c^2 = 0$
for the first of the simultaneous equations.

It is plausible that Einstein made a sign error here, initially getting $\frac{3\alpha^2}{4} + \frac{3\alpha^2}{2} + \lambda c^2 = 0$ for this equation. (W. Nahm)
A significant find

- **New perspective on steady-state theory (1950s)**
  Logical possibility: not a crank theory

- **Insight into Einstein’s philosophy**
  Discards model rather than introduce new term to GFE
  Occam’s razor approach

- **Insight into scientific progress**
  Unsuccessful theories important
  Understanding the development of successful theories
  Not Kuhnian paradigm shift
  Slow dawning

**Links with modern cosmology**
Dark energy: creation energy and $\lambda$
Cosmic inflation: de Sitter metric
Einstein's lost theory uncovered

Physicist explored the idea of a steady-state Universe in 1931.

Davide Castelvecchi

24 February 2014

New Discovery Reveals Einstein Tried To Devise A Steady State Model Of The Universe

Almost 20 years before the late Fred Hoyle and his colleagues devised the Steady State Theory, Albert Einstein toyed with a similar idea: that the universe was eternal, expanding outward with a consistent input of spontaneously generating matter.

An Irish physicist came across the paper last year and could hardly believe the model of the universe very different to today's Big Bang Theory.

The manuscript, which hadn't been referred to by scientists for decades, was carefully scanned and then uploaded to the internet.

WIT researchers discover ‘lost’ Einstein model of universe

Scientists uncovered mistitled papers while searching Jerusalem university's online archive.
The steady-state universe (1948)

- **Expanding but unchanging universe**
  - Hoyle, Bondi and Gold (1948)
  - Disliked speculation about physics of early epochs
  - Perfect cosmological principle?

- **Continuous creation of matter**
  - Very little matter required
  - No beginning, no age paradox

- **Replace λ with creation term** (Hoyle)
  \[
  G_{\mu\nu} + C_{\mu\nu} = k T_{\mu\nu}
  \]

- **Improved version (1962)**
  \[
  G_{\mu\nu} + \lambda g_{\mu\nu} = k T (C_\mu + C_\nu)
  \]
Steady-state vs big bang

- **Optical astronomy (1950s)**
  - Amended timescale of expansion (Baade, Sandage)
  - Age problem removed

- **Radio-astronomy (1960s)**
  - Galaxy distributions at different epochs
  - Cambridge 3C Survey (Ryle)

- **Cosmic microwave background**
  - Low temperature, low frequency
  - Remnant of early universe
Results: publications

- Einstein’s 1931 model

- Einstein’s steady-state manuscript

- Einstein-de Sitter model

- Review paper: conclusions
Einstein’s cosmology: conclusions

- **Major test for general relativity**
  - Assumptions; space-time = space + time
  - Homogeneous, isotropic and static universe

- **Embraces dynamic cosmology**
  - New evidence – new models (JMK)
  - Timespan of Friedman models puzzling
  - Steady-state universe?

- **Evolving models (less contrived)**
  - Simplest models first
  - Extraction of parameters; compatible with observation?
  - Timespan puzzle attributed to simplifying assumptions
  - No discussion of origins (wary of extrapolations)
  - Verdict (1933, 1945): more data needed

Hubble constant revised

Cosmic microwave background
Homogeneous, flat universe
Observational parameters needed (1930s)

- **Spatial curvature** \( k = -1, 0, 1 ? \)
- **Cosmic constant** \( \lambda = 0 ? \)
- **Deacceleration** \( q_0 = - \dot{R}/\ddot{R}^2 \)
- **Density of matter** \( \rho < \rho_{\text{crit}} ? \)
- **Timespan** \( \tau = 10^{10} \text{ yr} ? \)
- **Hubble constant** \( \dot{R}/R = 500 \text{ km s}^{-1}\text{Mpc}^{-1} ? \)

What do redshifts represent?
Is expansion a local effect?

Hubble and Tolman 1935
Einstein’s steady-state model and cosmology today

- **Dark energy (1998)**
  - Accelerated expansion (observation)
  - Positive cosmological constant

- **Einstein’s dark energy**
  - "The conservation law is preserved in that, by setting the \( \lambda \)-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1)."

- **Cosmic inflation**
  - Inflationary models use de Sitter metric
  - Used in all steady-state models
  - Flat curvature, constant rate of matter creation
  - Different time-frame!
Einstein’s steady-state theory: an abandoned model of the cosmos

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Received 1st February 2014 / Received in final form 12 May 2014
Published online (Inserted Last) © IOP Publishing, Springer-Verlag 2014

Abstract. We present a translation and analysis of an unpublished manuscript by Albert Einstein in which he attempted to construct a ‘steady-state’ model of the universe. The manuscript, which appears to have been written in early 1931, demonstrates that Einstein was considering a steady-state model in which the mass density of matter in an expanding universe is maintained constant by the continuous formation of matter from empty space. This model is very similar to previously revisited models.

Einstein’s cosmic model of 1931 revisited: an analysis and translation of a forgotten model of the universe

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Received: 21st September 2013 / Received in final form 20 December 2013
Published online 4 February 2014
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Abstract. We present an analysis and translation of Einstein’s 1931 paper “Zum kosmologischen Problem der allgemeinen Relativitätstheorie” or “On the cosmological problem of the general theory of relativity”. In this little-known paper, Einstein proposes a cosmic model in which the universe undergoes an expansion followed by a contraction, quite different to the traditionally-expanding Einstein-de Sitter model of 1932. The paper offers many insights into Einstein’s cosmology in the light of the first evidence for an expanding universe and we consider his views of issues such as the curvature of space, the cosmological constant, the similarity and the timescale of the expansion. A number of original
A cosmic puzzle

- **What is causing recession of the galaxies?**
  - If redshifts are velocities
  - If effect is non-local

- **Newton’s law of gravity**
  - Gravity pulls in, not out
  - No other long range force for neutral matter

- **Space, time are fixed**
  - Not affected by contents of universe
  - Eternal, infinite universe
Conclusions

**Cosmology – a testing ground for general relativity?**

Assumptions; space-time = space + time
Homogeneity and isotropy
Static universe

**Dynamic cosmology**

Steady-state universe?
Evolving models less contrived

**Evolving models**

Timespan problem: attributed to assumptions
Origins puzzle: ignored

**Verdict**

More data needed
An origin for the universe? (1931)

- Expanding $U$ smaller in the past
- Rewind expanding model to early epochs
- Extremely dense, extremely hot
- Expanding and cooling ever since
- Explosive beginning at $R = 0$?

Later called *The big bang*
Einstein’s steady-state model and cosmology today

- **Accelerated expansion (1998)**
  
  Supernova measurements
  
  *Dark energy – positive cosmological constant*

- **Einstein’s dark energy**
  
  “The conservation law is preserved in that, by setting the $\lambda$-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”

  *Anticipates positive cosmological constant*

- **De Sitter line element**
  
  $$ds^2 = -e^{at} (dx_1^2 + dx_2^2 + dx_3^2) + c^2 dt^2 \ldots$$

  *Necessary for all steady-state models*

  *Identical to inflationary models (different time-frame)*