The Big Bang: Fact or Fiction?

One Hundred Years of the Cosmological Constant

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Overview

I  Introducing the cosmological constant
   Einstein’s 1917 static model of the universe
   Problems of interpretation

II  The fallow years
    The expanding universe (1929)
    Abandoning the cosmological constant

III  Resurrection (1990s)
    The accelerating universe - dark energy
    Interpretation – the quantum energy of the vacuum
The field equations of GR (1915)

\[ G_{\mu\nu} = -\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \]

\[ G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu} \]

10 non-linear differential equations that relate the geometry of space-time to the density and flow of mass-energy

\[ ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \]

\[ ds^2 = \sum_{\mu,\nu=1}^{4} n_{\mu\nu} dx^\mu dx^\nu \]

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]

\[ ds^2 = \sum_{\mu,\nu=1}^{4} g_{\mu\nu} dx^\mu dx^\nu \]

\[ n_{\mu\nu} : \text{constants} \]

\[ g_{\mu\nu} : \text{variables} \]
Einstein’s 1917 model of the cosmos

- **Assume stasis** (*no evidence to the contrary*)
  Non-zero, uniform density of matter

- **Introduce closed spatial curvature**
  *To conform with Mach’s principle*
  Solves problem of $g_{\mu\nu}$

- **Introduce new term in GFE**
  *Allowed by relativity*
  Needed for non-zero solution

- **Quantitative model of the universe**
  Cosmic radius related to matter density
  Cosmic radius related to cosmological constant

\[
\Gamma_{\mu\nu} = -\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)
\]

\[
\Lambda_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)
\]

\[
\lambda = \frac{\kappa \rho}{2} = \frac{1}{R^2}
\]
Einstein’s view of the cosmological constant

**Not** an energy of space
Incompatible with Mach’s Principle

**A necessary evil**
“An ugly modification of the GFE” (Einstein 1918)

**Dispensable?**
Purpose = non-trivial static solution
“if the universe is not static, then away with the cosmic constant”

**A changing view**
New form of GFE (Einstein 1918)
$\lambda = \text{constant of integration}?$

Einstein’s postcard to Hermann Weyl
Einstein’s view of the cosmological constant

- **Introduced in analogy with Newtonian cosmology**
  
  Full section on Newtonian gravity (Einstein 1917)
  
  Indefinite potential at infinity?

- **Modifying Newtonian gravity at cosmic scales**
  
  Extra term in Poisson’s equation

- **A “foil” for relativistic models**
  
  Introduce cosmic constant in similar manner

- **Inexact analogy**
  
  Modified GFE corresponds to P3, not P2

- **A significant error?**
  
  Implications for interpretation

\[ \nabla^2 \phi = 4\pi G \rho \quad (P1) \]

\[ \nabla^2 \phi - \lambda \phi = 4\pi G \rho \quad (P2) \]

\[ G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \]

\[ \nabla^2 \phi + c^2 \lambda = 4\pi G \rho \quad (P3) \]
Schrödinger and the cosmological constant

**Schrödinger, 1918**

*Cosmic constant term not necessary for cosmic model*

*Negative pressure term in energy-momentum tensor*

**Einstein’s reaction**

*New formulation equivalent to original*

*(Questionable: physics not the same)*

**Schrödinger, 1918**

*Could pressure term be time-dependent?*

**Einstein’s reaction**

*If not constant, time dependence unknown*

*“I have no wish to enter this thicket of hypotheses”*

\[ T_{\mu\nu} = \begin{pmatrix} -p & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & \rho - p \end{pmatrix} \]

\[ g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu} \]
The stability of the Einstein World

- How does cosmic constant term work?
  Assume uniform distribution of matter

- Perturbation
  What happens if the density of matter varies slightly?

- Failed to investigate
  No mention of issue in 1917 paper
  No mention of issue until 1927, 1930

- Lemaître (1927)
  Cosmos expanding from Einstein World

- Eddington (1930)
  Einstein World unstable
Non-static cosmologies

Alexander Friedman (1922)

Consider time-varying solutions for the cosmos
Expanding or contracting universe
Retain cosmic constant for generality

Evolving universe

Time-varying radius and density of matter
Positive or negative spatial curvature
Depends on matter $\Omega = d/d_c$

Reception (1927)

Rejected by Einstein
Considered hypothetical (unrealistic)
Ignored by community

All possible universes
Lemaître’s universe (1927)

- **Georges Lemaître (1927)**
  - Allow time-varying solutions (expansion)
  - Retain cosmic constant

- **Compare astronomical observation**
  - Redshifts of the nebulae (Slipher)
  - Extra-galactic nature of the nebulae (Hubble)

- **Expansion from static Einstein World**
  - Instability (implicit)

- **Reception**
  - Ignored by community
  - Rejected by Einstein

  “Vôtre physique est abominable!”

\[
\frac{3 R'^2}{R^2} + \frac{3 c^2}{R^2} - \lambda = \kappa c^2 \rho, \\
\frac{R'^2}{R^2} + \frac{2 R''}{R^2} + \frac{c^2}{R^2} - \lambda = 0.
\]
The watershed: Hubble’s law

- **Hubble’s law (1929)**
  
  A linear redshift/distance relation for the spiral nebulae
  Linear relation: \( h = 500 \, \text{kms}^{-1}\text{Mpc}^{-1} \) (Cepheid I stars)

- **Evidence of cosmic expansion?**
  
  RAS meeting (1930): Eddington, de Sitter

- **Friedman-Lemaître models circulated**
  
  Time-varying radius and density of matter

- **Einstein apprised**
  
  Cambridge visit (June 1930)
  Sojourn at Caltech (Spring 1931)
Expanding models of the cosmos (1930 -)

- **Eddington (1930, 31)**
  
  *On the instability of the Einstein universe*
  Expansion caused by condensation?

- **Tolman (1930, 31)**
  
  *On the behaviour of non-static models*
  Expansion caused by annihilation of matter?

- **de Sitter (1930, 31)**
  
  *Further remarks on the expanding universe*
  Expanding universes of every flavour

- **Einstein abandons \( \lambda \) (1931, 32)**
  
  *Friedman-Einstein model* \( k = 1, \lambda = 0 \)
  *Einstein-de Sitter model* \( k = 0, \lambda = 0 \)

*Problem: Age paradox*
Abandoning the cosmic constant

- $\lambda$ used to address age problem
  - Eddington, Lemaitre, Tolman

- Resolution of the age problem
  - Recalibration of distance
  - Cepheid II stars; stellar intensities
  - New age $\sim 10^{10}$ years

- Cosmic constant abandoned
  - Unnecessary term

- Neglected for many years
  - Redundant 1950s-1990s

Walther Baade  
Allan Sandage
III The return of the cosmic constant

- New problems with expansion timeline (HST)
  *Resurrect* $\lambda$ – (Turner, Krauss, Carroll)

- Observations of Type Ia Supernovae (1998)
  *Light from furthest supernovae too dim*
  *Expansion speeding up in recent epochs?*

- Geometry of cosmos
  *Flat geometry - Astronomy, CMB*
  $\Omega = 1 \ (\text{but } \Omega_M = 0.3)$

*Dark energy*; extra component in energy density of universe
Modern big bang model: $\Lambda$-CDM

A flat, accelerating universe containing matter, dark matter and dark energy

1. Ordinary matter: 4% (astrophysics)

2. Dark matter: 22% (astrophysics)

3. Dark energy: 74% (supernova, CMB)

$$\Omega = 1 \quad (\Omega_M = 0.26; \ \Omega_\Lambda = 0.74)$$

*Einstein’s biggest blunder: setting the cc to zero?*
DE = quantum energy of the vacuum?

- Zero-point energy
  *Heisenberg Uncertainty Principle* \( \Delta t \Delta E \geq \hbar/2 \)

- Vacuum energy
  *Pure vacuum does not exist*
  *Particle pair production*

- Calculation on cosmic scales
  *Extremely large value*

- Incompatible with astronomical measurements of \( \Lambda \)

\[
\Omega_{\text{vac}} / \Omega_{\Lambda} = 10^{150}
\]
Summary

- Chequered history
  Introduction, abandonment and resurrection

- Undoubtedly back (DE)
  Cosmic acceleration (astronomy)
  Cosmic geometry

- Problem of interpretation
  Quantum energy of the vacuum
  Mismatch between theory and experiment

- Alternative explanations
  Quintessence
  Echo of cosmic inflation

*Alternative theories of gravity?*
Afterglow Light Pattern 400,000 yrs.

Dark Ages

Development of Galaxies, Planets, etc.

Dark Energy Accelerated Expansion

Inflation

Quantum Fluctuations

1st Stars about 400 million yrs.

Big Bang Expansion 13.7 billion years
The Friedman-Einstein model (1931)

- Cosmic constant abandoned
  - Unsatisfactory (unstable solution)
  - Unnecessary (non-static universe)

- Calculations of cosmic radius and density
  - Einstein: \( P \sim 10^8 \text{ lyr}, \rho \sim 10^{-26} \text{ g/cm}^3, \ t \sim 10^{10} \text{ yr} \)
  - We get: \( P \sim 10^9 \text{ lyr}, \rho \sim 10^{-28} \text{ g/cm}^3, \ t \sim 10^9 \text{ yr} \)

- Explanation for age paradox?
  - Assumption of homogeneity at early epochs

- Not a cyclic model
  - “Model fails at \( P = 0 \)”
  - Contrary to what is usually stated
The age paradox (1930-1950)

- Rewind Hubble graph (Lemaître 1931)
- $U$ smaller in the past
- Extremely dense, extremely hot (‘big bang’)
- Expanding and cooling ever since
- But time of expansion $= 1/H = 10^9$ years

*Universe younger than the stars?*
Lemaître’s universe (1931-33)

- Expansion from radioactive decay
- Retain cosmic constant
- Stagnant epoch
- Circumvents age problem
- Accelerated epoch

\[ \lambda = \text{Energy of vacuum} \]

\[ p = - \rho_0 c^2, \quad \rho_0 = \frac{\lambda c^2}{8\pi G} \]
“My greatest blunder”

- Einstein’s description of cosmic constant term
  *Reported by George Gamow*

- Controversy
  *Queried by Straumann, Livio
  Not in Einstein’s papers or other reports*

- Our findings
  *Consistent with actions
  Einstein’s remark reported by Gamow, Alpher, Wheeler*

- Meaning of remark
  *Failure to spot instability of static solution
  Failure to predict expanding universe*
WMAP Satellite (2002)

- Details of $CMB$ spectrum
- Details of galaxy formation
- Details of flatness of $U$
- Dark energy

Cosmic microwave background
WMAP measurements of CMB (2005)

- Spectrum of $T$ variations
- Geometry is flat (to 1%)
- Dark energy 74%

Strong support for dark energy

Strong support for inflation

Fit to theory
Measuring the cosmic constant

- **Calculate orbits of astronomical bodies**
  - Newtonian calculation

- **Compare with astronomical observation**
  - Difference = measure of cosmological constant
  - Possible in principle (Einstein 1921a)

- **Globular clusters**
  - Specific example (Einstein 1921b)
  - Null result

- **Future observations?**
  - More accurate data needed

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56. “A Simple Application of the Newtonian Law of Gravitation to Globular Star Clusters”

[(Einstein 1921)]

Published ca. 18 March 1921


There is hardly any doubt that one can safely extrapolate Newton’s law beyond the distances for which it has been verified. This confidence is also supported by the general theory of relativity, which provides a rational foundation for Newton’s law such that an extrapolation to bodies that interact over larger distances appears all the more justified. However, the general theory of relativity allows for considerable deviations from the Newtonian law in a spatially finite universe, yet only in the case where the mean density of the stellar matter in the investigated, gravitating

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Physics of dark energy

- Allowed by general relativity
- Cosmological constant term
  - Constant or variable?
- Natural tendency of space to expand?
- Quantum energy of vacuum?
  - *GR meets QT*
- Why of similar order to matter density?
- Conflict between theory and observation
- Other explanations for DE?
The general theory of relativity

- **The general principle of relativity (1907-)**
  Relativity and accelerated motion?

- **The principle of equivalence**
  Equivalence of gravity and acceleration

- **The principle of Mach**
  Relativity of inertia
  Structure of space determined by matter

- **A long road (1907-1915)**
  Gravity = curvature of space-time
  Covariant field equations?
From 3(a), in accordance with (1a) one calculates for the $R_{\mu\nu}$ 
$(x_1 = x_2 = x_3 = 0)$ the values
\[ \begin{array}{cccc}
-\frac{2}{p^2} & 0 & 0 & 0 \\
0 & -\frac{2}{p^2} & 0 & 0 \\
0 & 0 & -\frac{2}{p^2} & 0 \\
0 & 0 & 0 & 0 \\
\end{array} \]
for $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, the values
\[ \begin{array}{cccc}
\frac{1}{p^2} & 0 & 0 & 0 \\
0 & \frac{1}{p^2} & 0 & 0 \\
0 & 0 & \frac{1}{p^2} & 0 \\
0 & 0 & 0 & 0 \\
\end{array} \]
while for $-\kappa T$ one obtains the values
\[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\kappa \rho c^2 \\
\end{array} \]
Thus from (1) the two contradictory equations are obtained
\[ \begin{array}{c}
\frac{1}{p^2} = 0 \\
3c^2 \frac{1}{p^2} = \kappa \rho c^2 \\
\end{array} \tag{4} \]
\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu} \]
\[ ds^2 = \frac{dx_1^2 + dx_2^2 + dx_3^2}{\left(1 + \frac{r^2}{(2P)^2}\right)^2} - c^2 dt^2 \]
\[ \lambda \text{ term needed for (static) solution} \]
\[ \text{Interpretation?} \]

Einstein 1933
A natural progression

Ultimate test for new theory of gravitation

Principle 1: stasis
Assume static distribution of matter

Principle 2: uniformity
Assume no-zero, uniform distribution of matter

Principle 3: Mach’s principle
No such thing as empty space

Boundary conditions at infinity?
A spatially closed universe