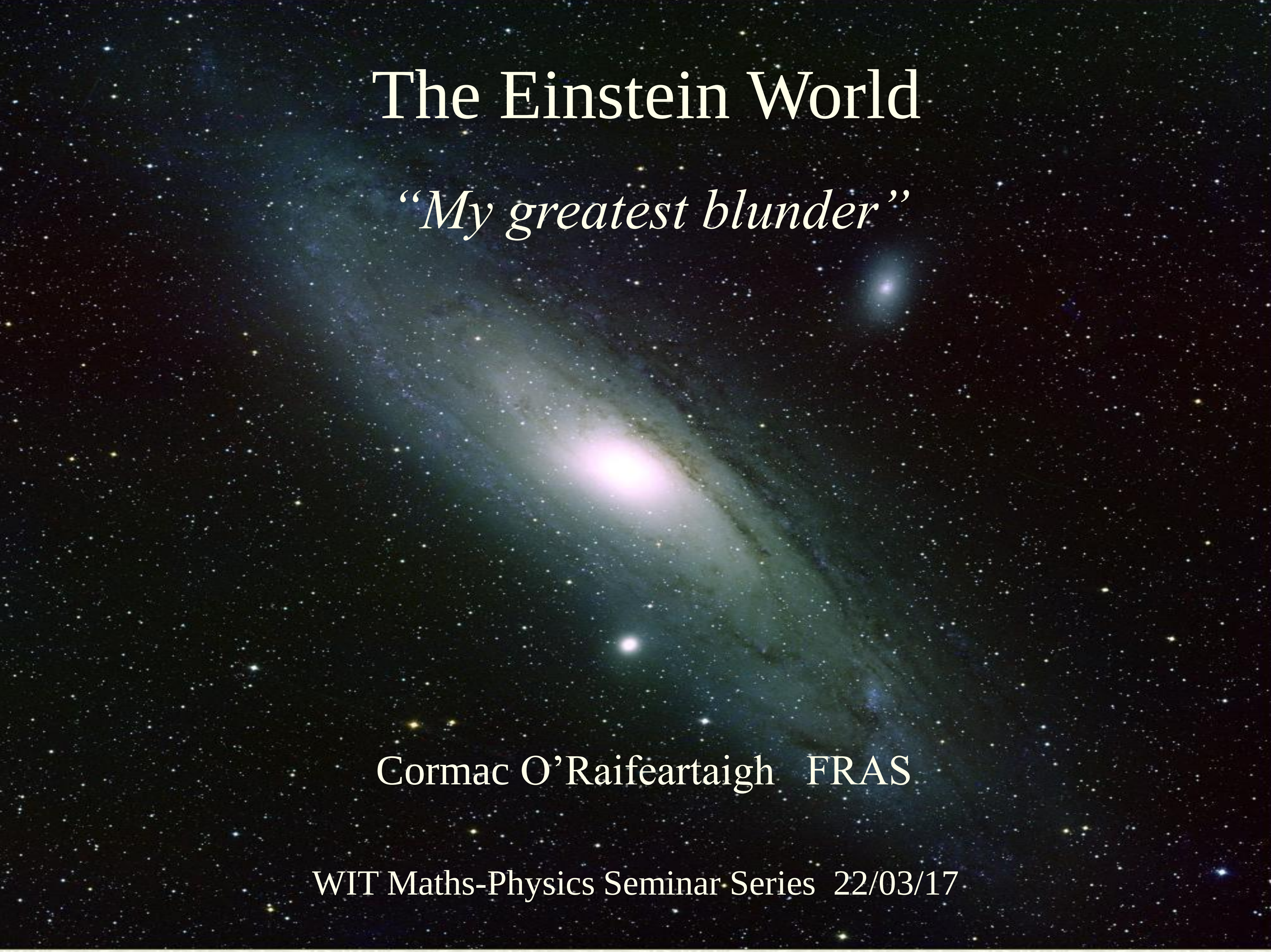


The Einstein World

"My greatest blunder"

Cormac O'Raifeartaigh FRAS

WIT Maths-Physics Seminar Series 22/03/17

Overview

- **The general theory of relativity (1916)**

  A new theory of gravity, space and time

- **General relativity and the universe (1917)**

  Third phase of the relativity project

- **The Einstein World (1917)**

  Assumptions: characteristics

  The cosmological constant

  Issues of size and stability

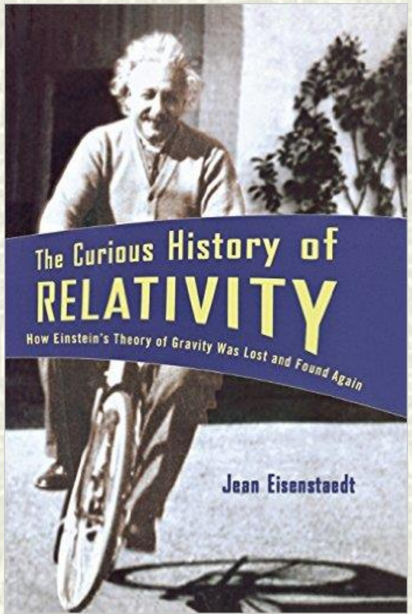
- **The expanding universe**

  Hubble's law

  "My greatest blunder"

Coda: The Einstein World today

Einstein in Berlin (1916)



Biographical considerations

- **Appointed to Berlin Chair**
  Arrives April 1914
  Mileva leaves June 1914

- **World War I (1914-18)**
  Living alone, food shortages
  Dietary problems, illness

- **Extremely productive period**
  Covariant general theory of relativity (1915)
  Exposition, solutions and predictions (1916)
  First relativistic model of the cosmos (1917)
  Papers on gravitational waves
  Papers on the quantum theory of radiation
  Papers on unified field theory

Einstein in Berlin (1916)

The general theory of relativity (1916)

- **Extending the special theory (1907-)**

  Relativity and accelerated motion?

  Relativity and gravity?

- **<u>The principle of equivalence</u>**

  Equivalence of gravity and acceleration

  Consequence of Galileo's principle

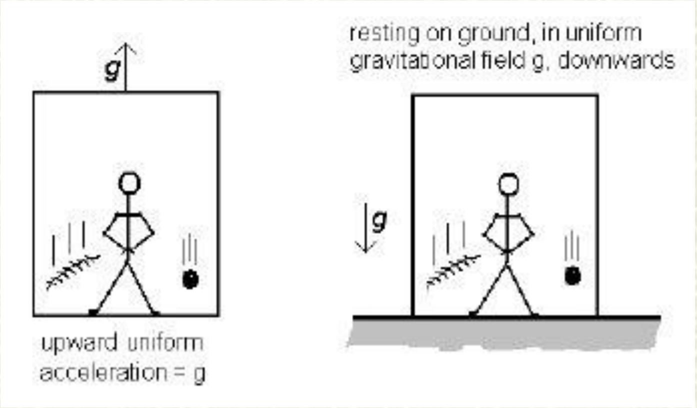
- **<u>The principle of Mach</u>**

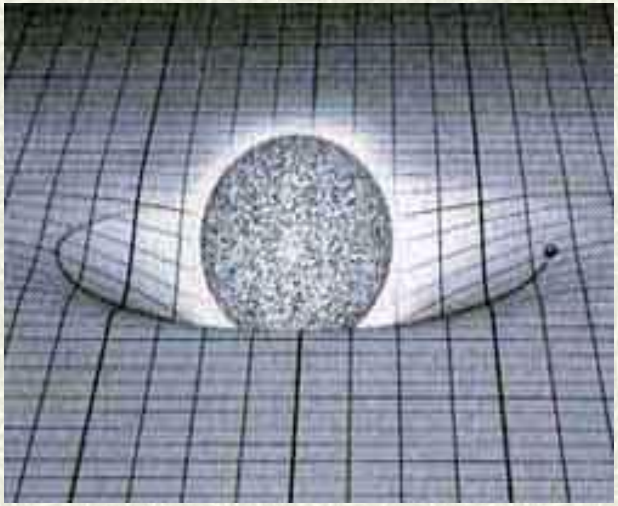
  Inertia defined by matter

- **A long road (1907-1915)**

  Space-time defined by matter

  Gravity = curvature of space-time



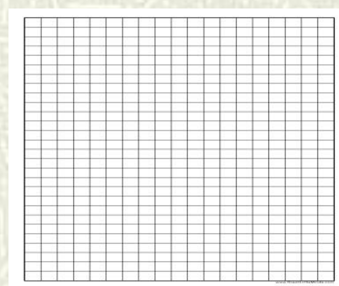


The field equations of GR

$$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

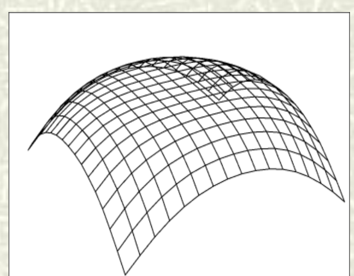
10 non-linear differential equations that relate the geometry of space-time to the density and flow of mass-energy

SR $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$



GR $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$

$$ds^2 = \sum_{\mu,\nu=1}^{4} n_{\mu\nu} dx^\mu dx^\nu$$



$$ds^2 = \sum_{\mu,\nu=1}^{4} g_{\mu\nu} dx^\mu dx^\nu$$

$n_{\mu\nu}$: *constants*

$g_{\mu\nu}$: *variables*

Applying GR to the universe

- **Apply GR to the Universe (Einstein, 1917)**

  Ultimate test for new theory of gravitation

- **Principle 1: stasis**

  Assume static distribution of matter

- **Principle 2: uniformity**

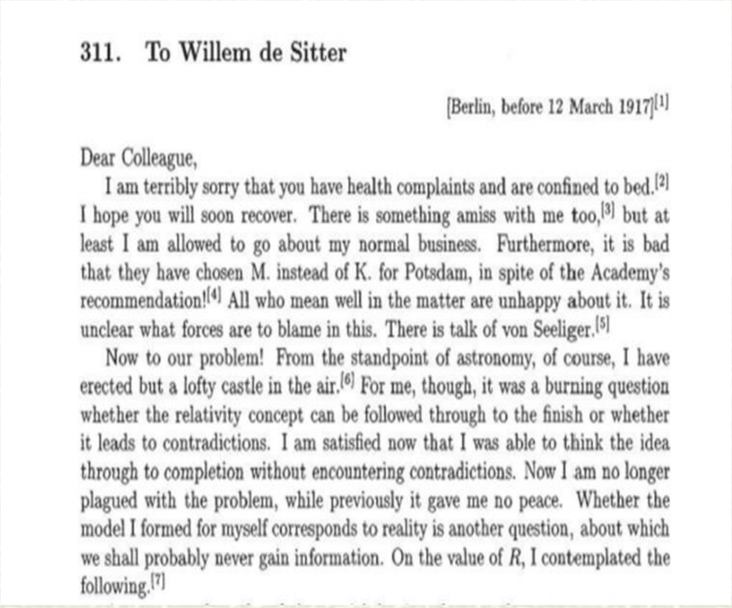
  Assume uniform distribution of matter

- **Principle 3: relativity of inertia**

  No such thing as empty space

- **Boundary conditions at infinity?**

  What are the 'natural' values of the $g_{\mu\nu}$?



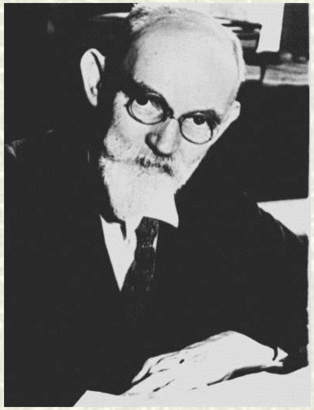
311. To Willem de Sitter

[Berlin, before 12 March 1917][1]

Dear Colleague,

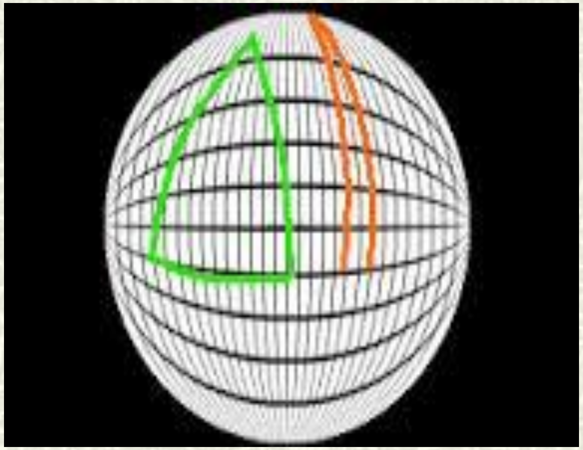
I am terribly sorry that you have health complaints and are confined to bed.[2] I hope you will soon recover. There is something amiss with me too,[3] but at least I am allowed to go about my normal business. Furthermore, it is bad that they have chosen M. instead of K. for Potsdam, in spite of the Academy's recommendation![4] All who mean well in the matter are unhappy about it. It is unclear what forces are to blame in this. There is talk of von Seeliger.[5]

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]



The Einstein World

- **Assume stasis**
  No evidence to the contrary
  Einstein's model of the Static Universe



- **Assume closed spatial curvature**
  To conform with Mach's principle
  Banish the boundaries!

$$ds^2 = \frac{dx_1^2 + dx_2^2 + dx_3^2}{\left(1 + \frac{r^2}{(2P)^2}\right)^2} - c^2 dt^2$$

$$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

- **The price**
  New term needed in field equations
  *The cosmological constant **

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$$

- **Quantitative model of the universe**
  Radius related to matter density
  Radius related to cosmic constant

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

On the cosmological constant

From 3(a), in accordance with (1a) one calculates for the $R_{\mu\nu}$ $(x_1 = x_2 = x_3 = 0)$ the values

$$\begin{matrix} -\frac{2}{P^2} & 0 & 0 & 0 \\ 0 & -\frac{2}{P^2} & 0 & 0 \\ 0 & 0 & -\frac{2}{P^2} & 0 \\ 0 & 0 & 0 & 0, \end{matrix}$$

for $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, the values

$$\begin{matrix} \frac{1}{P^2} & 0 & 0 & 0 \\ 0 & \frac{1}{P^2} & 0 & 0 \\ 0 & 0 & \frac{1}{P^2} & 0 \\ 0 & 0 & 0 & -\frac{3c^2}{P^2}, \end{matrix}$$

while for $-\kappa \boldsymbol{T}$ one obtains the values

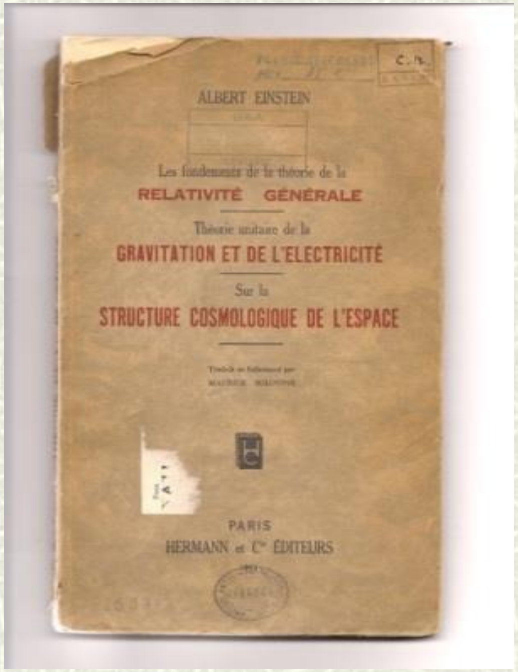
$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\kappa\rho c^2 \end{matrix}$$

Thus from (1) the two contradictory equations are obtained

$$\left.\begin{aligned} \frac{1}{P^2} &= 0 \\ \frac{3c^2}{P^2} &= \kappa\rho c^2 \end{aligned}\right\} \quad (4)$$

$$\boldsymbol{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa \; T_{\mu\nu}}$$

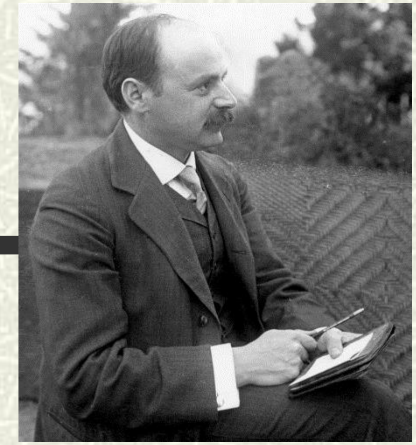
$$ds^2 = \frac{dx_1^2 + dx_2^2 + dx_3^2}{\left(1 + \frac{r^2}{(2P)^2}\right)^2} - c^2 dt^2$$



Einstein 1933

λ needed for non-zero static solution

Curved universes before 1917



Karl Schwarzschild
(1873–1916)

- **Non-Euclidean geometry (19th century)**

  The geometry of Riemann

  The geometry of Lobachevsky and Bolai

- **The curved universe**

  The universe of Lobachevsky

  The closed universe of Zöllner

- **The Schwarzschild universe**

  The closed universe of Schwarzschild (1900)

  Constraints from considerations of parallax

- **An intriguing anticipation**

  Schwarzschild's 1916 letter to Einstein

  Einstein's response

Esteemed Mr. Einstein,

Many thanks for your letter of January 9th.[1] I wrote about Jupiter to Hertzsprung,[2] who immediately brought my attention to the fact that in the next few years Jupiter will be orbiting too much to the south of us. This extreme precision is only attainable between the zenith and perhaps a declination of 50°. The problem must therefore be left to observatories more to the south. Mr. Freundlich could be of service if he were to pick out stellar transits and occultations. (It appears to me, though, that Mr. Banachiewicz[3] has already been engaged in this for other purposes.) In other respects, we shall not be able to agree too easily on Freundlich, and I only want to add: Debating this way and that about him is pointless. I just think that he has already fallen out with Struve to such a degree that it would be best if you exerted your influence toward obtaining another occupation for him.[4]

With regard to the inertial system, we are in agreement. You say that beyond the Milky Way conditions could exist under which the Galilean system is no longer the simplest. I only contend that within the Milky Way such conditions do not exist. As far as very large spaces are concerned, your theory takes an entirely similar position to Riemann's geometry, and you are certainly not unaware that elliptic geometry is derivable from your theory, if one has the entire universe under uniform pressure (energy tensor $-p, -p, -p, 0$).[5]

I cannot deny that you have put the freedom extending beyond it to most fortunate use.

Structure of Einstein's 1917 paper

§ 1. The Newtonian Theory

§ 2. The Boundary Conditions According to the General Theory of Relativity

§ 3. The Spatially Finite Universe with a Uniform Distribution of Matter

§ 4. On an Additional Term for the Field Equations of Gravitation

§ 5. Calculation and Result

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

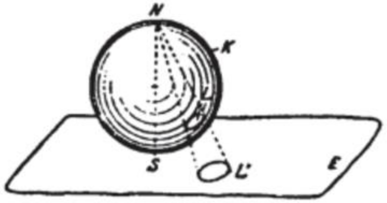


FIG. 2

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen hinzutreten für das räumlich Unendliche, falls man die Welt wirklich als räumlich unendlich ausgedehnt anzusehen hat.

Bei der Behandlung des Planetenproblems habe ich diese Grenzbedingungen in Gestalt folgender Annahme gewählt: Es ist möglich, ein Bezugssystem so zu wählen, daß sämtliche Gravitationspotentiale $g_{\mu\nu}$ im räumlich Unendlichen konstant werden. Es ist aber a priori durchaus nicht evident, daß man dieselben Grenzbedingungen ansetzen darf, wenn man größere Partien der Körperwelt ins Auge fassen will. Im folgenden sollen die Überlegungen angegeben werden, welche ich bisher über diese prinzipiell wichtige Frage angestellt habe.

§ 1. Die Newtonsche Theorie.

Es ist wohlbekannt, daß die Newtonsche Grenzbedingung des konstanten Limes für ϕ im räumlich Unendlichen zu der Auffassung hinführt, daß die Dichte der Materie im Unendlichen zu null wird. Wir denken uns nämlich, es lasse sich ein Ort im Weltraum finden, um den herum das Gravitationsfeld der Materie, im großen betrachtet, Kugelsymmetrie besitzt (Mittelpunkt). Dann folgt aus der Poissonschen Gleichung, daß die mittlere Dichte ρ rascher als $\frac{1}{r^2}$ mit wachsender Entfernung r vom Mittelpunkt zu null herabsinken muß, damit ϕ im

Doc. 43

Cosmological Considerations in the General Theory of Relativity [1]

This translation by W. Perrett and G. B. Jeffery is reprinted from H. A. Lorentz et al., *The Principle of Relativity* (Dover, 1952), pp. 175–188.

It is well known that Poisson's equation

$$\nabla^2\phi = 4\pi K\rho \qquad (1)$$

in combination with the equations of motion of a material point is not as yet a perfect substitute for Newton's theory of action at a distance. There is still to be taken into account the condition that at spatial infinity the potential ϕ tends toward a fixed limiting value. There is an analogous state of things in the theory of gravitation in general relativity. Here, too, we must supplement the differential equations by limiting conditions at spatial infinity, if we really have to regard the universe as being of infinite spatial extent. [2]

In my treatment of the planetary problem I chose these limiting conditions in the form of the following assumption: it is possible to select a system of reference so that at spatial infinity all the gravitational potentials $g_{\mu\nu}$ become constant. [3] But it is by no means evident *a priori* that we may lay down the same limiting conditions when we wish to take larger portions of the physical universe into consideration. In the following pages the reflexions will be given which, up to the present, I have made on this fundamentally important question.

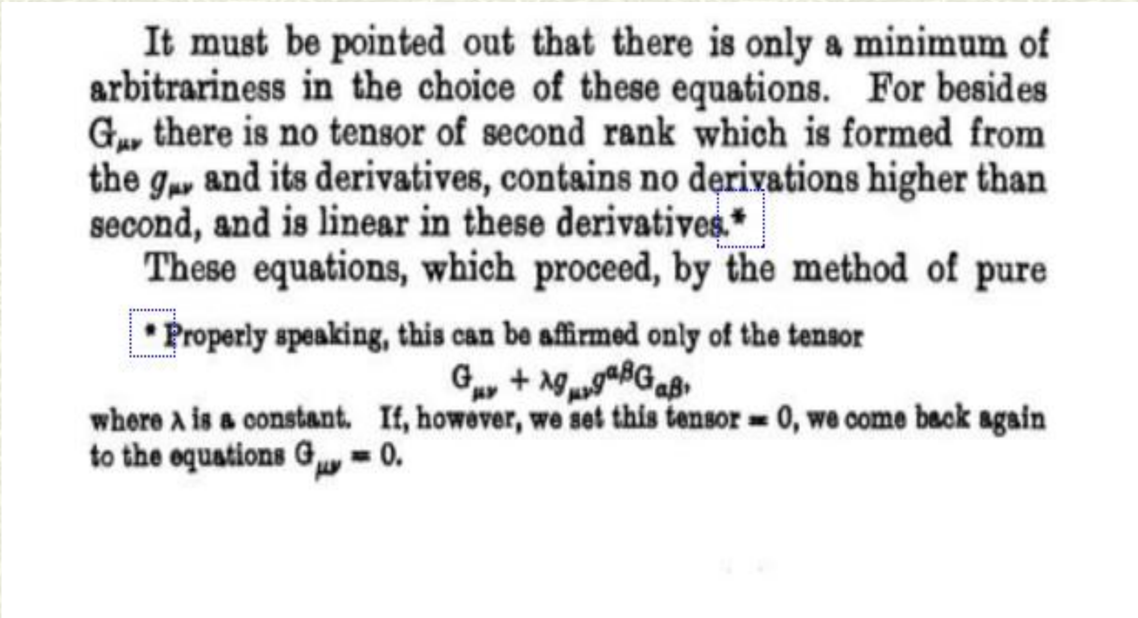
§ 1. The Newtonian Theory

It is well known that Newton's limiting condition of the constant limit for ϕ at spatial infinity leads to the view that the density of matter becomes zero at infinity. For we imagine that there may be a place in universal space round about which the gravitational field of matter, viewed on a large scale, possesses spherical symmetry. It then follows from Poisson's equation that, in order that ϕ may tend to a

On the cosmological constant (i)

- Introduced in 1916. *Ann. Physik.* **49:** 769-822

  Die Grundlage der allgemeinen Relativitätstheorie



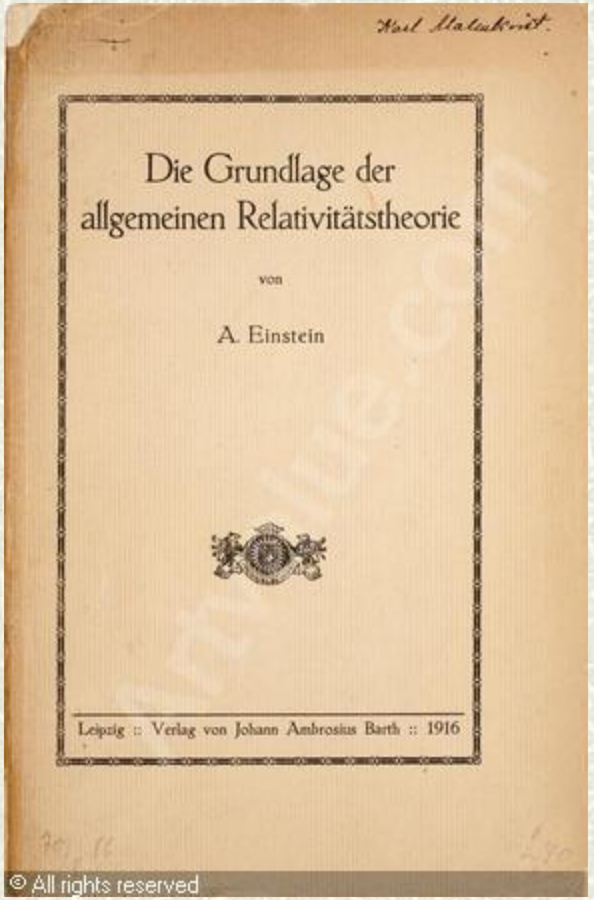
It must be pointed out that there is only a minimum of arbitrariness in the choice of these equations. For besides $G_{\mu\nu}$ there is no tensor of second rank which is formed from the $g_{\mu\nu}$ and its derivatives, contains no derivations higher than second, and is linear in these derivatives.*

These equations, which proceed, by the method of pure

* Properly speaking, this can be affirmed only of the tensor

$$G_{\mu\nu} + \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta},$$

where λ is a constant. If, however, we set this tensor $= 0$, we come back again to the equations $G_{\mu\nu} = 0$.



- Contains an error: div $(\lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta}) \neq 0$

On the cosmological constant (ii)

- **Introduced in analogy with Newtonian cosmology**

  Full section on Newtonian gravity (Einstein 1917)

  Indefinite potential at infinity?

$$\nabla^2\phi = 4\pi G\rho \quad \text{(P1)}$$

$$\nabla^2\phi - \lambda\phi = 4\pi G\rho \quad \text{(P2)}$$

- **Modifying Newtonian gravity**

  Extra term in Poisson's equation

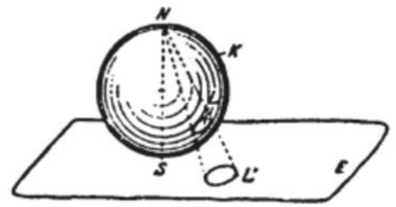


FIG. 2

- **A "foil" for relativistic models**

  Introduce cosmic constant in similar manner

- **<u>Inexact analogy</u>**

  Modified GFE corresponds to P3, not P2

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$$

- **A significant error?**

  Implications for interpretation

$$\nabla^2\phi + c^2\lambda = 4\pi G\rho \quad \text{(P3)}$$

On the cosmological constant (iii)

- **Schrödinger, 1918**

  Cosmic constant not necessary for cosmic model

  New term in matter-energy tensor (RHS)

- **Einstein's reaction**

  New formulation equivalent to original

  (Questionable: physics not the same)

- **Schrödinger, 1918**

  Could cosmological term be time-dependent ?

- **Einstein's reaction**

  If not constant, time dependence unknown

  "I have no wish to enter this thicket of hypotheses"

Erwin Schrödinger 1887-1961

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$$

$$G_{\mu\nu} = \lambda g_{\mu\nu} - \kappa T_{\mu\nu}$$

The size of the Einstein World

- **What is the size of the Einstein World?**

  Assume uniform distribution of matter

- **Density of matter**

  Density of matter in MW from astronomy

  Assume density MW = density of cosmos?

- **Failed to calculate**

  No estimate of size in 1917 paper

  Declares density unknown in reviews

- **Calculation in correspondence!**

  Takes $\rho = 10^{-22}$ g/cm^3 $\rightarrow$ $R = 10^7$ light-years

  Compares unfavourably with 10^4 light-years (astronomy)

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]

Astronomers have found the spatial density of matter from star counts up to the nth size class, fairly independent of the class to which the count extends, at about

$$10^{-22}\,\mathrm{g/cm^3}.$$

From this, approximately

$$R = 10^7 \text{ light-years}$$

results, whereas we only see as far as 10^4 light-years. One thing seems strange to me, though. Stars close to our antipodal point should be emitting a lot of light to us.[8] It is doubtful, however, that they could appear point-shaped, since the light velocity varies irregularly. If such a thing were visible in the heavens, it would be noticeable through its negative parallax.[9] We should at least keep an eye out whether any objects with a negative parallax exist in the sky. But now, enough of this, or else you will laugh at me.

EINSTEIN CANNOT MEASURE UNIVERSE

With Mean Density of Matter Unknown the Problem Is Impossible.

FINAL PRINCETON LECTURE

Universe Called Finite and Yet Infinite Because of Its Curved Nature.

The stability of the Einstein World

- **How does cosmic constant work?**

  Assume uniform distribution of matter

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

- **Perturbation**

  What happens if the density of matter varies slightly?

- **Failed to investigate**

  No mention of issue in 1917

  No mention of issue until 1927, 1930

- **Lemaître (1927)**

  Cosmos expanding from Einstein World

- **Eddington (1930)**

  Einstein World unstable

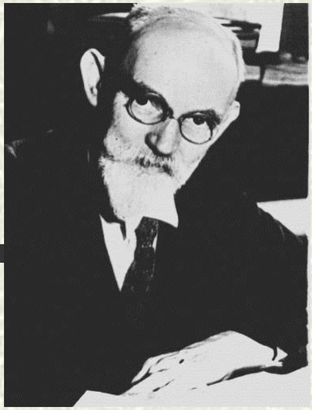
668 *Prof. A. S. Eddington,* xc. 7,

On the Instability of Einstein's Spherical World.
By A. S. Eddington, F.R.S.

1. Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein's spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître * which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies. Although not expressly stated, it is at once apparent from his formulæ that the Einstein world is unstable—an important fact which, I think, has not hitherto been appreciated in cosmogonical discussions. Astronomers are deeply interested in these recondite problems owing to their connection with the behaviour of spiral nebulæ; and I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître's brilliant solution.

Finitude of space depends on a "cosmical constant" λ, which occurs in Einstein's gravitational equations $G_{\mu\nu} = \lambda g_{\mu\nu}$ for empty space. On general philosophical grounds † there can be little doubt that this form of the equations is correct rather than his earlier form $G_{\mu\nu} = 0$; but λ is so small as to be negligible in all but very large scale applications. Except in so far as a value may be suggested by astronomical survey of the extragalactic universe, λ is unknown; or it would be better to say that we do not know the lengths of the objects and standards of our

The de Sitter universe (1917)



- **Alternative cosmic solution for the GFE**
  A universe empty of matter (1917)
  Closed curvature of space-time

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 0$$

$$\rho = 0: \ \lambda = \frac{3}{R^2}$$

- **Solution B**
  Solution enabled by cosmic constant
  Curvature of space determined by cosmic constant

- **Einstein's reaction**
  Unrealistic
  Conflict with Mach's principle

- **Interest from astronomers**
  Light from star would be redshifted
  Chimed with astronomical measurements

Nov. 1917. *Einstein's Theory of Gravitation.* 3

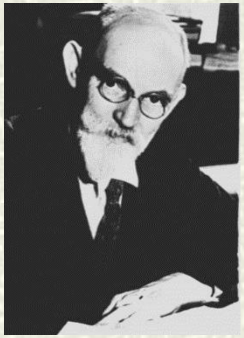
On Einstein's Theory of Gravitation, and its Astronomical Consequences. Third Paper.* By W. de Sitter, Assoc. R.A.S.

Contents of Third Paper.

1. On the relativity of inertia. New form of the field-equations. Two solutions A and B of these equations.
2. On space with constant positive curvature. Comparison of the two systems A and B.
3. Rays of light and parallax in the two systems. Hyperbolical space.
4. Motion of a material particle in the inertial field of the two systems. Further comparison of the two systems.
5. Differential equations for the gravitational field of the sun. Approximate integration of these equations.
6. Estimates of R in the system A.
7. Estimates of R in the system B.

1. In Einstein's theory of general relativity there is no essential difference between gravitation and inertia. The combined effect of the two is described by the fundamental tensor $g_{\mu\nu}$, and how much of it is to be called inertia and how much gravitation is entirely arbitrary. We might abolish one of the two words, and

The Einstein-de Sitter debate



- **de Sitter solution disliked by Einstein**

  Conflict with Mach's principle

  Problems with singularities? (1918)

- **Never accepted as realistic model**

  Lack of singularity conceded

  Considered unrealistic

  $$\rho = 0: \ \lambda = \frac{3}{R^2}$$

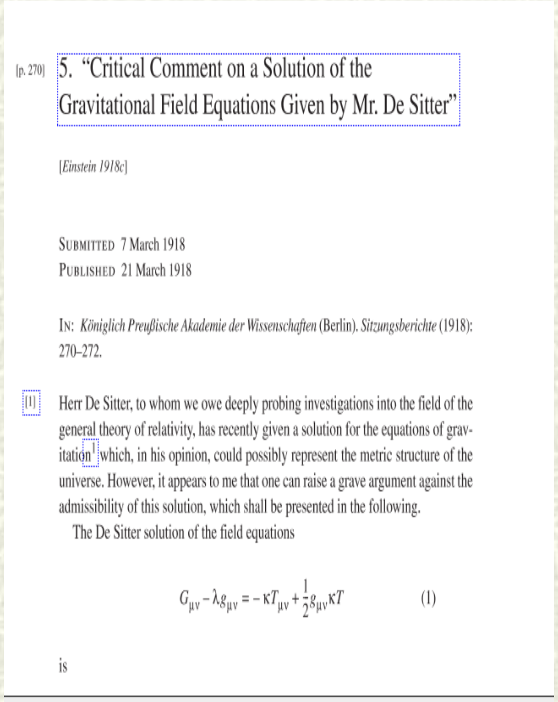
  Not static?

- **The de Sitter confusion**

  Static or non-static - a matter of co-ordinates?

  Weyl, Lanczos, Klein, Lemaître

Redshift prediction – astronomical interest



[p. 270] 5. "Critical Comment on a Solution of the Gravitational Field Equations Given by Mr. De Sitter"

[*Einstein 1918c*]

SUBMITTED 7 March 1918
PUBLISHED 21 March 1918

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1918): 270–272.

[1] Herr De Sitter, to whom we owe deeply probing investigations into the field of the general theory of relativity, has recently given a solution for the equations of gravitation[1] which, in his opinion, could possibly represent the metric structure of the universe. However, it appears to me that one can raise a grave argument against the admissibility of this solution, which shall be presented in the following.

The De Sitter solution of the field equations

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \kappa T \qquad (1)$$

is

The dynamic universe (theory)

- **Alexander Friedman (1922)**

  Allow time-varying solutions for the cosmos

  Two differential equations for R

$$\frac{3R'^2}{R^2}+\frac{3c^2}{R^2}-\lambda=\kappa c^2\rho,$$

$$\frac{R'^2}{R^2}+\frac{2RR''}{R^2}+\frac{c^2}{R^2}-\lambda=0.$$

Alexander Friedman (1888 -1925)

- **Evolving universe**

  Time-varying radius and density of matter

  Considered 'suspicious' by Einstein

$$\frac{1}{c^2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2=\frac{A-R+\frac{\lambda}{3c^2}R^3}{R}$$

Georges Lemaître (1894-1966)

- **Georges Lemaître (1927)**

  Theoretical universe of time-varying radius

  Expanding universe in agreement with emerging astronomical data

  Also rejected by Einstein

"Vôtre physique est abominable"

The watershed

Edwin Hubble (1889-1953)

- **Hubble’s law (1929)**

  A redshift/distance relation for the galaxies

  Linear relation: h = 500 $kms^{-1}Mpc^{-1}$

- **Evidence of cosmic expansion?**

  RAS meeting (1930): Eddington, de Sitter

- **Friedman-Lemaître models circulated**

  Time-varying radius and density of matter

- **Einstein apprised**

  Cambridge visit (June 1930)

  Sojourn at Caltech (Spring 1931)

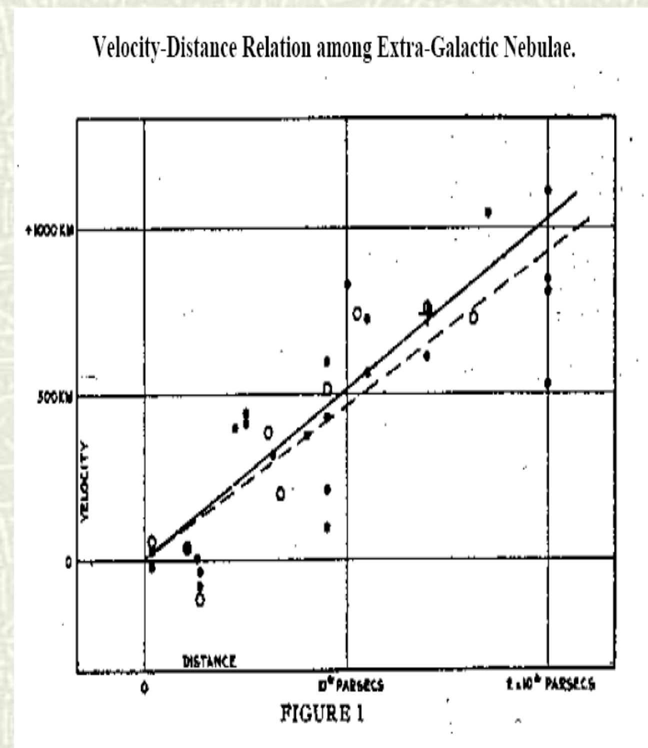


FIGURE 1

The expanding universe (1930 -)

- **Eddington (1930, 31)**

  On the instability of the Einstein universe
  Expansion caused by condensation?

- **Tolman (1930, 31)**

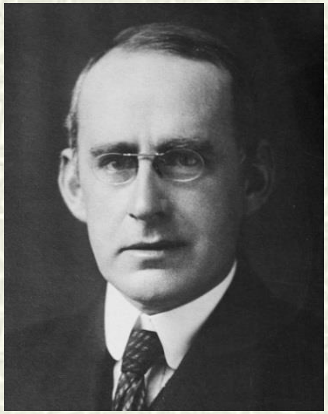
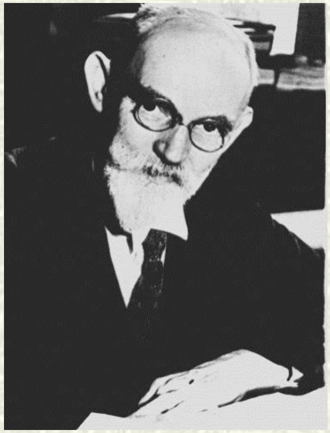
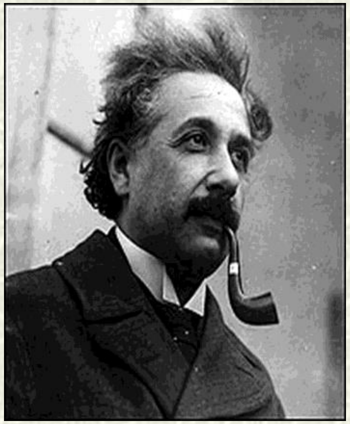
  On the behaviour of non-static models
  Expansion caused by annihilation of matter ?

- **de Sitter (1930, 31)**

  Further remarks on the expanding universe
  Expanding universes of every flavour

- **Einstein (1931, 32)**

  Friedman-Einstein model $k = 1$, $\lambda = 0$
  Einstein-de Sitter model $k = 0$, $\lambda = 0$

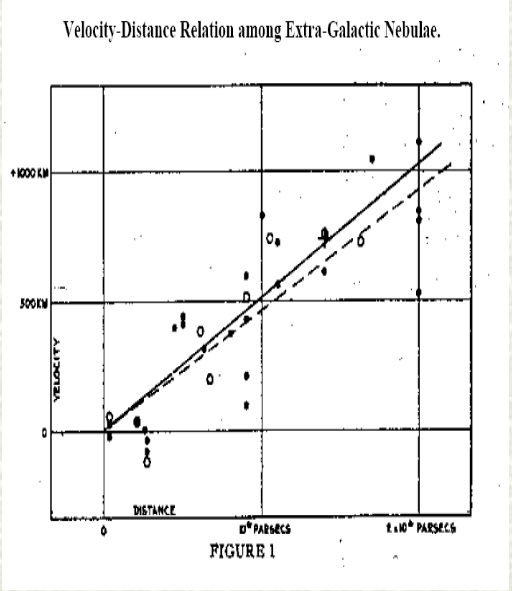





Expanding models
No mention of origins

Cosmic prediction: the big bang

- **Lemaître 1931**: expanding *U* smaller in the past
- Extrapolate to very early epochs
- Extremely dense, extremely hot
- Expanding and cooling ever since
- 'Fireworks beginning' at $R = 0$?

Fr Georges Lemaître

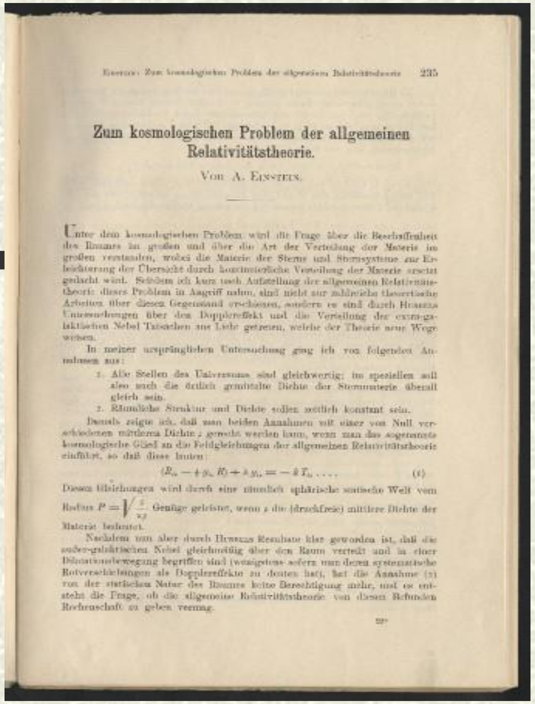


Not endorsed by Einstein, community (1930-60)

Simplified models: timescale problem

Later called ***'The big bang'***

Einstein's 1931 model (*F-E*)



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Zum kosmologischen Problem der allgemeinen Relativitätstheorie.

Von A. Einstein.

- **Einstein's first expanding model**

  Rarely cited (not translated)

$$\frac{3P'^2}{P^2} + \frac{3c^2}{P^2} - \lambda = \kappa c^2 \rho \,.$$

$$\frac{P'^2}{P^2} + \frac{2P''}{P} + \frac{c^2}{P^2} - \lambda = 0$$

- **Adopts Friedman 1922 model**

  Instability of static solution

  Hubble's observations

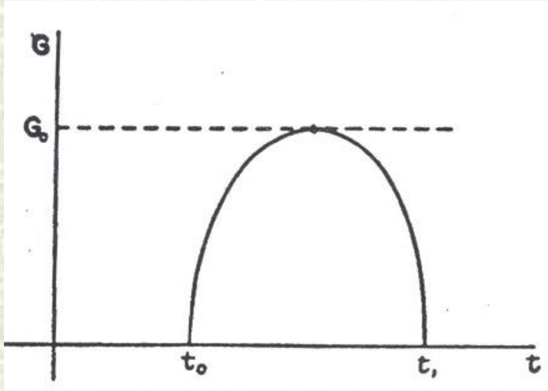
$$\left(\frac{dP}{dt}\right)^2 = c^2 \frac{P_0 - P}{P}$$

$$D = \frac{1}{P}\frac{dP}{dt}\cdot\frac{1}{c}$$

- **Sets cosmic constant to zero**

  Unsatisfactory, redundant

$$D^2 = \frac{1}{P^2}\frac{P_0 - P}{P} \qquad P \sim \frac{1}{D}$$



- **Extraction of cosmic parameters**

  $P \sim 10^8$ lyr *: $\rho \sim 10^{-26}$* g/cm^3

  $t \sim$ 10^{10} yr *: conflict with astrophysics*

  Attributed to simplifying assumptions (homogeneity)

$$D^2 = \frac{1}{3}\kappa\rho\frac{P_0 - P}{P} \qquad D^2 \sim \kappa\rho$$

Einstein's 1931 model revisited

- **First translation into English**

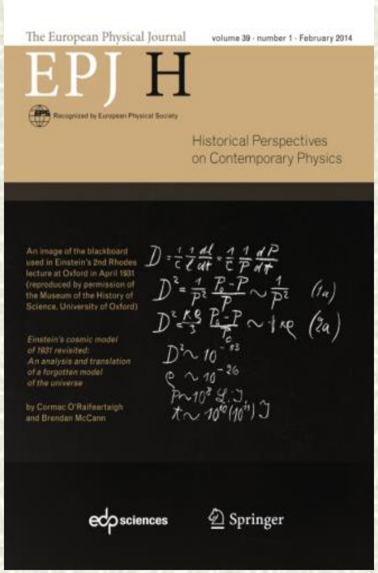
  O'Raifeartaigh and McCann 2014

- <u>**Not a cyclic model**</u>

  "Model fails at P = 0 "

  Contrary to what is usually stated

  $P \sim \frac{1}{D}$ $D^2 \sim \kappa\rho$



- **Anomalies in calculations of radius and density**

  Einstein: $P \sim 10^8$ lyr, $\rho \sim 10^{-26}$ g/cm^3, $t \sim 10^{10}$ yr

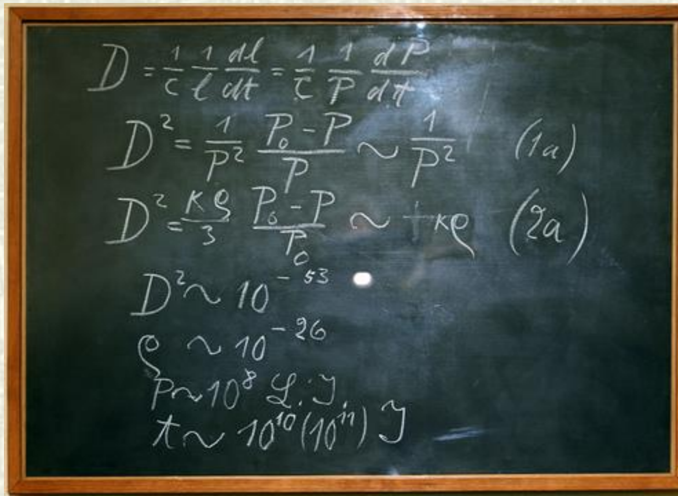
  We get: $P \sim 10^9$ lyr, $\rho \sim 10^{-28}$ g/cm^3, $t \sim 10^9$ yr

- **Source of error?**

  Oxford blackboard: $D^2 \sim 10^{-53}$ cm^{-2} *should be* 10^{-55} cm^{-2}

  Time miscalculation $t \sim 10^{10}$ yr (*should be* 10^9 yr)

  Non-trivial error: misses conflict with radioactivity



Einstein-de Sitter model (1932)

- **Curvature not a given in dynamic models**

  Not observed empirically

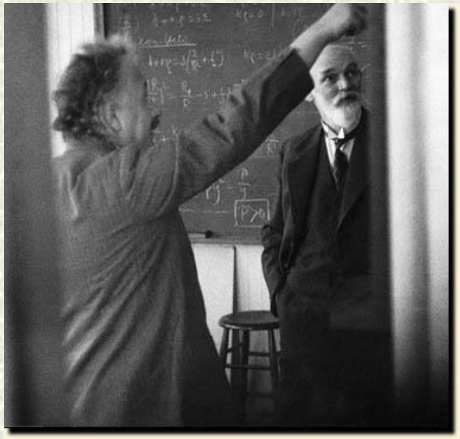
  Remove spatial curvature (Occam's razor)

$$ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2dt^2$$

$$\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,$$

$$\frac{1}{R^2}\left(\frac{dR}{cdt}\right)^2 = \frac{1}{3}\kappa\rho.$$

$$h^2 = \frac{1}{3}\kappa\rho$$



- **Simplest Friedman model**

  Time-varying universe with $\lambda = 0$, $k = 0$

  Important hypothetical case: critical universe

  Critical density : $\rho = 10^{-28}$ g/cm^3

- **<u>Becomes standard model</u>**

  Despite high density of matter

  Despite age problem

<u>Time evolution not considered in paper – see title</u>

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 18 March 15, 1932 Number 3

ON THE RELATION BETWEEN THE EXPANSION AND THE MEAN DENSITY OF THE UNIVERSE

By A. Einstein and W. de Sitter

Communicated by the Mount Wilson Observatory, January 25, 1932

In a recent note in the *Göttinger Nachrichten*, Dr. O. Heckmann has pointed out that the non-static solutions of the field equations of the general theory of relativity with constant density do not necessarily imply a positive curvature of three-dimensional space, but that this curvature may also be negative or zero.

Einstein-de Sitter model revisited

- **Einstein's cosmology review of 1933**

  Review of dynamic models from first principles

  Cosmic constant banished

  *Curved **or** flat geometry*

$$2A\frac{d^2A}{dt^2} + \left(\frac{dA}{dt}\right)^2 = 0$$

$$3\left(\frac{\frac{dA}{dt}}{A}\right)^2 = \varkappa\rho c^2.$$

- **Parameters extracted**

  Critical density of 10^{-28} g/cm^3 : reasonable

  <u>Timespan of 10^{10} years</u>: conflict with astrophysics

  Attributed to simplifications (incorrect estimate)

$$3h^2 = \varkappa\rho c^2 \; (= 8\pi K\rho)$$

$$A = c(t - t_0)^{\frac{2}{3}}.$$

- **Published in 1933!**

  French book; small print run

  Intended for scientific journal; not submitted

  Significant paper

$$t - t_0 = \frac{2}{3h}.$$

“My greatest blunder”

- **Einstein’s description of cosmic constant**

  Reported by George Gamow

- **Controversy**

  Queried by Straumann, Livio

  Not in Einstein’s papers or other reports

- **Our findings**

  Einstein’s remark reported by Gamow, Alpher, Wheeler

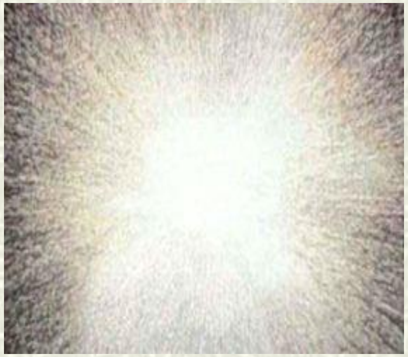
  Consistent with actions

- **Meaning of remark**

  Failure to spot instability of Einstein World

  Failure to predict expanding universe

Georges Gamow



The cosmic constant today

- **An accelerated expansion**

  Supernova measurements of galaxies

- **Confirmation**

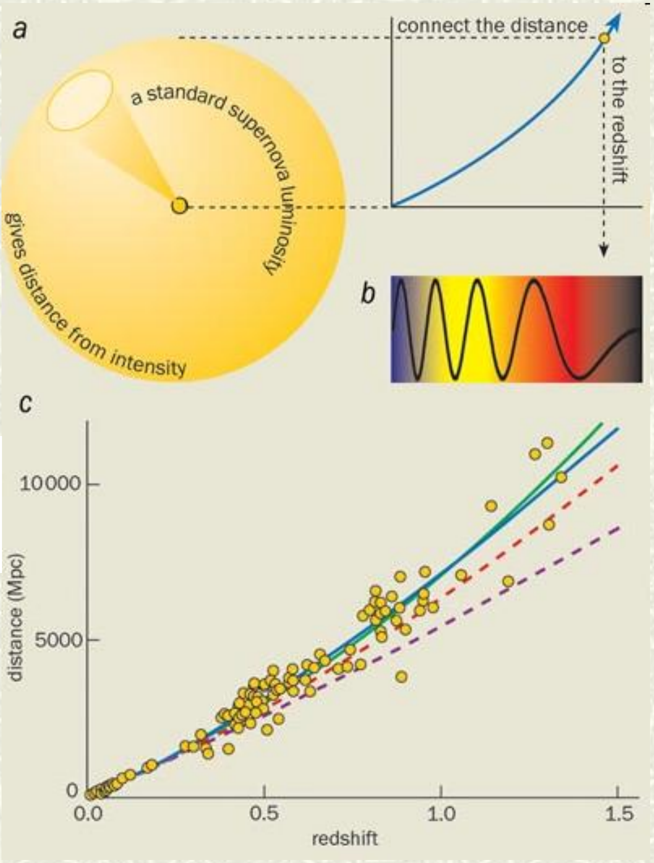
  Structure of cosmic microwave background

- **Dark energy**

  What is pushing out?

  Link to vacuum energy?

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$$



- **A new role for the cosmic constant**

  An idea whose time has come

Einstein's biggest blunder: <u>removing</u> the cc ?

The Einstein World: Summary

- **Apply GR to the universe**

  February 1917

- **Assumed static universe**

  The Known Universe

  Closed curvature (Machian)

- **Cosmological constant added**

  Allowed by relativity

  Analogy with Newtonian cosmology

- **The expanding universe (observation)**

  Cosmological constant removed

The Static Universe

Einstein's greatest blunder?

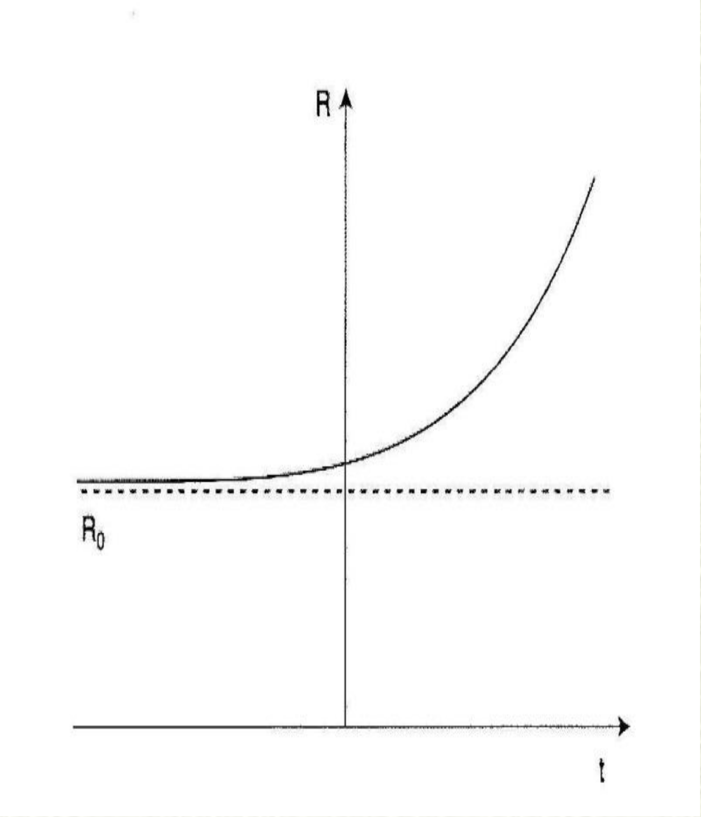
Coda: The Einstein World Today

- **An expanding universe**
  The recession of the galaxies

- **The question of origins**
  The problem of the singularity

- **<u>The emergent universe</u>**
  Inflating from a static state
  The Einstein World

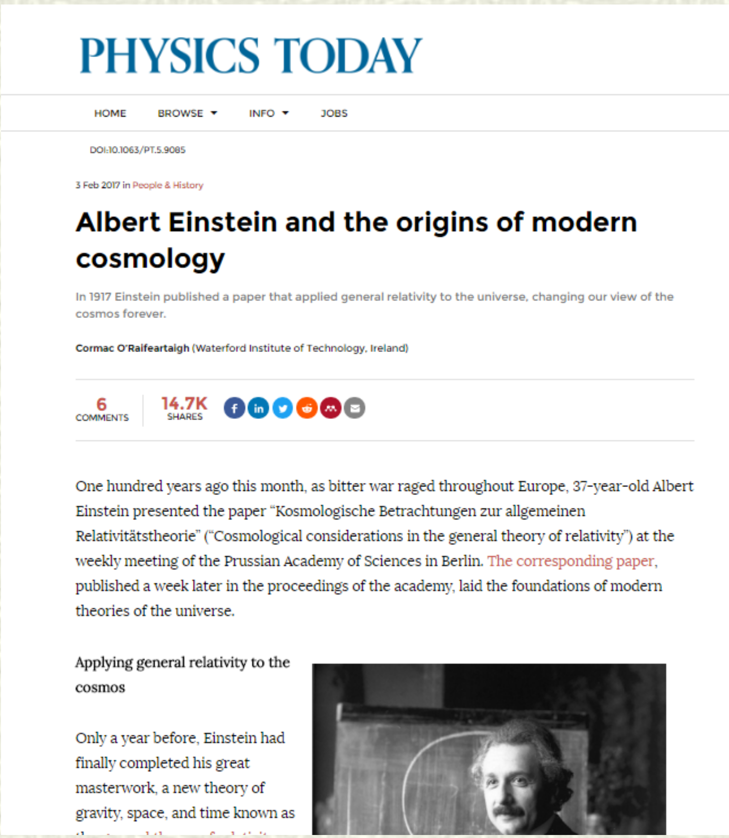
- **Stabilizing the Einstein World**
  Advanced GR: LQG, DGR, B-D, f(R), f (R,T)



Relevance of past models of the cosmos

Publications

Einstein's 1917 Static Model of the Universe: A Centennial Review
Accepted for publication in *Eur. Phys. J. (H)*



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DOI:10.1063/PT.5.9085

3 Feb 2017 in People & History

Albert Einstein and the origins of modern cosmology

In 1917 Einstein published a paper that applied general relativity to the universe, changing our view of the cosmos forever.

Cormac O'Raifeartaigh (Waterford Institute of Technology, Ireland)

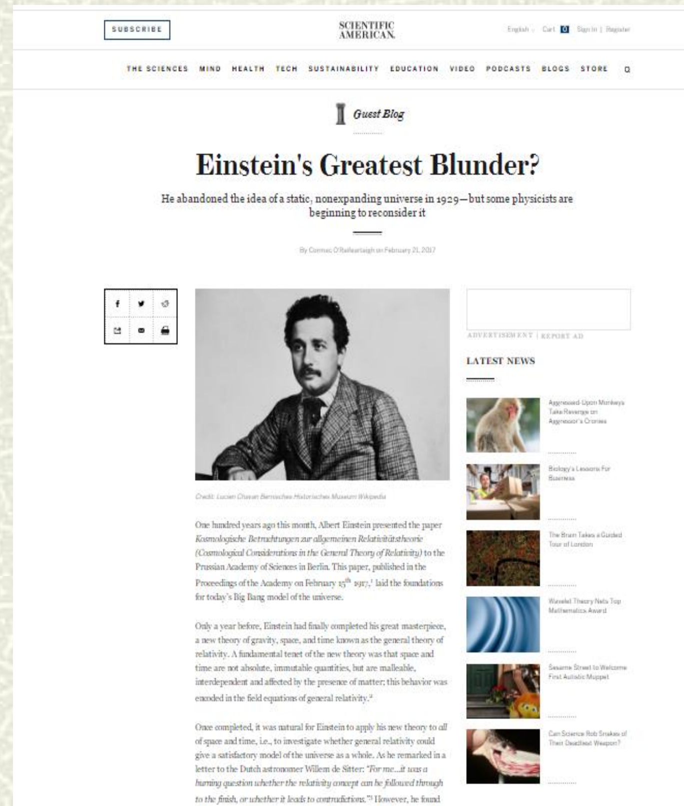
6 COMMENTS | 14.7K SHARES

One hundred years ago this month, as bitter war raged throughout Europe, 37-year-old Albert Einstein presented the paper "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie" ("Cosmological considerations in the general theory of relativity") at the weekly meeting of the Prussian Academy of Sciences in Berlin. The corresponding paper, published a week later in the proceedings of the academy, laid the foundations of modern theories of the universe.

Applying general relativity to the cosmos

Only a year before, Einstein had finally completed his great masterwork, a new theory of gravity, space, and time known as

Physics Today, Feb. 2017



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Guest Blog

Einstein's Greatest Blunder?

He abandoned the idea of a static, nonexpanding universe in 1929—but some physicists are beginning to reconsider it

By Cormac O'Raifeartaigh on February 21, 2017

Credit: Lucien Chavan Bernisches Historisches Museum Wikipedia

One hundred years ago this month, Albert Einstein presented the paper *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (Cosmological Considerations in the General Theory of Relativity)* to the Prussian Academy of Sciences in Berlin. This paper, published in the Proceedings of the Academy on February 15th 1917,[1] laid the foundations for today's Big Bang model of the universe.

Only a year before, Einstein had finally completed his great masterpiece, a new theory of gravity, space, and time known as the general theory of relativity. A fundamental tenet of the new theory was that space and time are not absolute, immutable quantities, but are malleable, interdependent and affected by the presence of matter; this behavior was encoded in the field equations of general relativity.[2]

Once completed, it was natural for Einstein to apply his new theory to *all* of space and time, i.e., to investigate whether general relativity could give a satisfactory model of the universe as a whole. As he remarked in a letter to the Dutch astronomer Willem de Sitter: "*For me...it was a burning question whether the relativity concept can be followed through to the finish, or whether it leads to contradictions.*"[3] However, he found

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Can Science Rob Snakes of Their Deadliest Weapon?

Scientific American, Feb. 2017

Physics > History and Philosophy of Physics

Einstein's 1917 Static Model of the Universe: A Centennial Review

Cormac O'Raifeartaigh, Michael O'Keeffe, Werner Nahm, Simon Mitton

(Submitted on 25 Jan 2017)

We present a historical review of Einstein's 1917 paper 'Cosmological Considerations in the General Theory of Relativity' to mark the centenary of a key work that set the foundations of modern cosmology. We find that the paper followed as a natural next step after Einstein's development of the general theory of relativity and that the work offers many insights into his thoughts on relativity, astronomy and cosmology. Our review includes a description of the observational and theoretical background to the paper; a paragraph-by-paragraph guided tour of the work; a discussion of Einstein's views of issues such as the relativity of inertia, the curvature of space and the cosmological constant. Particular attention is paid to little-known aspects of the paper such as Einstein's failure to test his model against observation, his failure to consider the stability of the model and a mathematical oversight in his interpretation of the role of the cosmological constant. We discuss the insights provided by Einstein's reaction to alternate models of the universe proposed by Willem de Sitter, Alexander Friedman and Georges Lema\^itre. Finally, we consider the relevance of Einstein's static model of the universe for today's 'emergent' cosmologies.

Comments: Submitted to the European Physical Journal (H).70-page review, 4 figures
Subjects: **History and Philosophy of Physics (physics.hist-ph)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)
Cite as: arXiv:1701.07261 **[physics.hist-ph]**
(or **arXiv:1701.07261v1 [physics.hist-ph]** for this version)