Overview

100 years of general relativity
   The special theory of relativity
   The general theory of relativity

General relativity and the universe
   The expanding universe
   The big bang model

Einstein’s universe
   Some new findings
First formulation of the equations of GR (1915)

First full formulation of the theory of GR (1916)

Die Feldgleichungen der Gravitation.
Von A. Einstein.

In zwei vor kurzen erschienenen Mitteilungen habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Formulations beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender: Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Nähierung enthalten und beliebigen Substitutionen von der Determinante $\gamma$ gegenüber kovariant waren. Hierauf fand ich, dass diese Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der Matie entweder. Das Koordinatensystem war dann nach der einfachsten Regel zu spezialisieren, dass $\sqrt{-g}$ zu $\gamma$ gemacht wird. Wodurch die Gleichungen der Theorie eine enge Vereinfachung erfahren. Dabei musste aber, wie erwähnt, die Hypothese eingeführt werden,
The special theory of relativity

- **Two new principles (1905)**
  
  Laws of physics identical for observers in uniform motion
  
  Speed of light identical for observers in uniform motion

- **Implications**
  
  Distance and time not absolute
  
  Distance and time not identical for bodies in uniform motion

- **Predictions for high-speed bodies**
  
  Length contraction; time dilation
  
  Mass increase; mass-energy equivalence \( E = mc^2 \)

- **Space + time = spacetime**

\[ ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \]
Evidence for special relativity

- **Invariance of the speed of light**
  
  *Always measured as* $c$

- **Time dilation**
  
  *The long-lived muon*

- **Particle experiments at the LHC**
  
  *Maximum velocity =* $c$
  
  *Mass increase*
  
  *Particle creation* $E = mc^2$
The general theory of relativity (1916)

- **General relativity**
  
  *Relativity and accelerated motion?*
  
  *Relativity and gravity?*

- **The equivalence principle**
  
  *No distinction between inertial and gravitational mass*

- **A new theory (1916)**
  
  *Space-time distorted by mass*
  
  *Gravity = curvature of space-time*

  \[ G_{\mu\nu} = -\kappa T_{\mu\nu} \]

- **Evidence**
  
  *Orbit of Mercury*
  
  *Bending of starlight by the sun (Eddington, 1919)*
Evidence for general relativity

- **Bending of distant light by stars**
  Gravitational lensing

- **Gravitational time dilation**
  GPS corrections

- **Black holes**
  \[ G_{\mu\nu} = - \kappa T_{\mu\nu} \]
  Centre of galaxies

- **Gravitational waves**
  *Hulse-Taylor* (1979): energy loss in binary pulsar
  *LIGO* (2016); gravitational waves
II  Relativity and the universe

Apply general relativity to the cosmos (1917)
Ultimate test for new theory of gravity

Dynamic universe?
Expanding or contracting

Static universe (observation)
Add new term to field equations
Cosmological constant \( \lambda \)

Bounded universe
Cosmic radius and matter density defined by \( \lambda \)
No empty space at infinity (Mach’s principle)
De Sitter’s universe (1917)

Alternative static solution

‘Empty’ universe

\[ G_{\mu\nu} + \lambda g_{\mu\nu} = 0 \]

Disliked by Einstein

Conflict with Mach’s principle

Beginning of Einstein’s dislike for cosmic constant

Interest from astronomers

Prediction of redshifts – Slipher effect?

Confusion: static or non-static model?

Weyl 1923, Lanczos 1923, Lemaître 1925
Friedman’s universe

- Allow time-varying solutions (1922)
  
  Closed spatial curvature
  
  \[ G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu} \]

- Expanding or contracting universe

- Badly received
  
  Considered “suspicious” by Einstein
  
  “To this a physical reality can hardly be ascribed”

- Further Friedman models (1924)
  
  Negative spatial curvature
  
  Cosmic evolution, geometry depends on matter content
Lemaître’s universe (1927)

- Expanding model of the cosmos from GR

- Compare with astronomy
  - Redshifts of the spiral nebulae (Slipher)
  - Distances of the spiral nebulae (Hubble)
  - Evidence of cosmic expansion?

- Ignored by community
  - Belgian journal (in French)
  - Einstein: “Votre physique est abominable”
The watershed (1929)

- Spiral nebulae far beyond Milky Way
  
  *A universe of galaxies (1925)*

- A linear relation between redshift and distance: Hubble’s law (1929)
  
  \[ v = H_0 d \quad \text{with} \quad H = 500 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \]

- Landmark result in astronomy

  *Far-away galaxies rushing away at a speed proportional to distance*
RAS meeting (1930)
If redshifts are velocities, and if effect is non-local Static cosmic models don’t match observations

Expanding universe?
Hubble’s law = expansion of space?
\[ H = 500 \text{ kms}^{-1}\text{Mpc}^{-1} \]

Friedman-Lemaître model circulated
Time-varying radius
Evolving models
Time-varying density of matter
Models of the expanding universe

- **Eddington (1930, 31)**
  
  *On the instability of the Einstein universe*
  
  Expansion caused by condensation?

- **Tolman (1930, 31)**
  
  *On the behaviour of non-static models*
  
  Expansion caused by annihilation of matter?

- **de Sitter (1930, 31)**
  
  *Further remarks on the expanding universe*
  
  Expanding universes of every flavour

- **Einstein (1931, 32)**
  
  *Friedman-Einstein model* \( \lambda = 0, \ k = 1 \)
  
  *Einstein-de Sitter model* \( \lambda = 0, \ k = 0 \)

  *New: also a steady -state model*

\[
G_{\mu\nu} = -\kappa T_{\mu\nu}
\]
The big bang model (1931)

- Expanding $U$ smaller in the past
- Rewind to early epochs
- Extremely dense, extremely hot
- Explosive beginning at $R = 0$?
- Expanding and cooling ever since

Later called ‘The big bang’
A second line of evidence

- **How were the chemical elements formed?**  
  *In the stars? Why so much Hydrogen?*

- **Elements formed in the hot big bang?**  
  *Georges Gamow*  
  *Predicts $U = 75\%$ Hydrogen, 25\% Helium*

- **Agreement with observation**  
  *Victory for big bang model*  
  *(Heavier atoms formed in stars)*
A third line of evidence

- Infant universe very hot indeed
  Dominated by radiation

- Still observable today?
  A fossil from the early universe

- The cosmic microwave background
  Low temp, microwave frequency
  Released when atoms formed (300,000 yr)

No-one looked (1940s)
Steady-state universe (1948)

- Alternative to big bang
  Hoyle, Bondi and Gold

- Expanding, unchanging universe?

- Continuous creation of matter
  Very little matter needed

- No assumptions about early epochs
Steady-state vs big bang

- **Radio-astronomy** (1950s, 1960s)
  Galaxy distributions at different epochs
  Evidence of evolution

- **Quasars, pulsars** (1960s)

- **Cosmic microwave background** (1965)
  Low temperature, low frequency
  Remnant of young, hot universe

**Strong support for big bang model**

Martin Ryle

Penzias and Wilson (1965)
Modern measurements of the CMB

- Details of background radiation
- Full spectrum
- Comparison with theory
- Perturbations?

- Ground telescopes
- Balloon experiments
- Satellite experiments

COBE satellite (1992)
COBE measurements of CMB

- Expected temperature
- Expected frequency
- Perfect blackbody spectrum

- Radiation very uniform
- Variation of 1 in $10^5$
- Seeds of galaxies?

Nobel Prize

COBE (1992)
The big bang – evidence

1. The expansion of the $U$

2. The abundance of $H$ and $He$

3. The distribution of the galaxies

4. The cosmic microwave background

How did it start?
III Einstein’s 1931 model

- **Einstein’s first dynamic model of the cosmos**
  Rarely cited (German)

- **Adopts Friedman 1922 model**
  *Time-varying, closed universe: k = 1*
  Cosmic constant redundant: set $\lambda = 0$

- **Use Hubble to extract empirical parameters**
  *Radius $R \sim 10^8$ lyr, timespan $10^{10}$ yr*
  *Density of matter $\rho \sim 10^{-26}$ g/cm$^3$*

- **Not a cyclic model**
Einstein’s 1931 model (research)

- First translation into English
  
  O’Raifeartaigh and McCann 2014

- Anomalies in calculations
  
  $R \sim 10^8$ lyr : should be $10^9$ lyr
  
  $\rho \sim 10^{-26}$ g/cm$^3$ : should be $10^{-28}$ g/cm$^3$
  
  $t \sim 10^{10}$ yr : should be $10^9$ yr

- Error in Hubble constant
  
  Oxford blackboard: $D^2 \sim 10^{-53}$ cm$^{-2}$

- Implications for timespan problem
Finding 2: Einstein’s steady-state model

- Unpublished manuscript
  Archived as draft of Einstein’s 1931 model

- Steady-state model
  Expanding model of constant density
  “The density is constant and determines the expansion”

- Continuous creation
  Continuous formation of matter from vacuum
  Associated with cosmic constant

- Fatal flaw: abandoned
  Evolving models adopted instead
Abandoned model

**Correct geometry**
- de Sitter line element

**Simultaneous equations**
- Error in derivation
- Null solution

**Einstein’s crossroads**
- Realised problem on revision
- Declined to amend model

**Evolving models**
- Less contrived and set $\lambda = 0$
Einstein’s lost theory uncovered

Physicist explored the idea of a steady-state Universe in 1931.

Davide Castelvecchi

24 February 2014

New Discovery Reveals Einstein Tried To Devise A Steady State Model Of The Universe

Almost 20 years before the late Fred Hoyle and his colleagues devised the Steady State Theory, Albert Einstein toyed with a similar idea: that the universe was eternal, expanding outward with a consistent input of spontaneously generating matter.

An Irish physicist came across the paper last year and could hardly believe model of the universe very different to today’s Big Bang Theory. The manuscript, which hadn’t been referred to by scientists for decades.
100 years of general relativity

Published May 1916

*From Swiss patent office to Berlin*

A new theory of gravity

*Gravity = curvature of spacetime*

Predictions supported by experiment

*Bending of light by a star*

*Expanding universe*

*Gravitational time dilation (GPS)*

One more test

*Gravitational waves - LIGO 2016!*

Einstein in Berlin (1918)
Einstein’s universe: conclusions

- **Cosmology** = test for general relativity
  Introduces cosmic constant

- **Embraces dynamic cosmology**
  New evidence – new models
  Steady-state model
  Evolving models simpler: remove cosmic constant

- **The evolving universe**
  Extract observational parameters
  Timespan problem attributed to simplifying assumptions

- **No discussion of origins**
  Wary of extrapolations to early epochs

Hubble constant revised
Dark energy (1998)

Accelerated expansion (observation)
Positive cosmological constant

Einstein’s dark energy

“The conservation law is preserved in that, by setting the $\lambda$-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”

Cosmic inflation

Inflationary models use de Sitter metric
Used in all steady-state models
Flat curvature, constant rate of matter creation
Different time-frame!
Einstein’s steady-state model: key quotes

New solution

“In what follows, I wish to draw attention to a solution to equation (1) that can account for Hubbell’s facts, and in which the density is constant over time”

Matter creation

“If one considers a physically bounded volume, particles of matter will be continually leaving it. For the density to remain constant, new particles of matter must be continually formed within that volume from space “

Dark energy

“The conservation law is preserved in that, by setting the $\lambda$-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”
Einstein-de Sitter model (1932)

Curvature not a given in dynamic models
Not observed empirically
Remove spatial curvature (Occam’s razor)

\[ ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2 dt^2 \]

Simplest Friedman model
Time-varying universe with \( \lambda = 0, k = 0 \)
Important hypothetical case: critical universe
Critical density: \( \rho = 10^{-28} \text{ g/cm}^3 \)

Becomes standard model
Despite high density of matter
Despite age problem

Time evolution not considered in paper – see title
Einstein-de Sitter model revisited

- **Einstein’s cosmology review of 1933**
  - Review of dynamic models from first principles
  - Culminates in Einstein-de Sitter model
  - Cosmic constant banished
  - Possibility of flat geometry

- **Parameters extracted**
  - Critical density of $10^{-28}$ g/cm$^3$: reasonable
  - Timespan of $10^{10}$ years: conflict with astrophysics
  - Attributed to simplifications (incorrect estimate)

- **Published in 1933!**
  - French book; small print run
  - Intended for scientific journal; not submitted
  - Significant paper
Einstein’s steady-state theory: a significant find?

- **New perspective on steady-state theory (1950s)**
  
  Logical possibility: not a crank theory

- **Insight into scientific progress**
  
  Evolution of successful theories
  
  No Kuhnian paradigm shift to ‘big bang’ model
  
  Slow dawning

- **Insight into Einstein’s philosophy**
  
  Simple solution?

  Discards model rather than introduce new term to GFE

  Occam’s razor approach

- **Links with modern cosmology**
  
  Dark energy, cosmic inflation

  *Paradigm shift or slow dawning?*
Explanation for runaway galaxies?

Newton

- Gravity pulls in not out
- Space is fixed
- Time has no beginning

How can galaxies be receding? What is pushing out?
Results: publications

- **Einstein’s 1931 model**
  

- **Einstein’s steady-state manuscript**


- **Einstein-de Sitter model**


- **Review paper: conclusions**
Einstein’s cosmic model of 1931 revisited: an analysis and translation of a forgotten model of the universe

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Abstract. We present an analysis and translation of Einstein’s 1931 paper ‘‘Zeitlich geometrische Probleme der allgemeinen Relativitätstheorie’’ or ‘‘On the cosmological problem of the general theory of relativity.’’ In this little-known paper, Einstein proposes a cosmic model in which the universe undergoes an expansion followed by a contraction, quite different to the continually-expanding Einstein-de Sitter model of 1932. The paper offers many insights into Einstein’s cosmology for the light of the first evidence for an expanding universe and we consider his views of forces such as the curvature of space, the cosmological constant, the similarity and the timeliness of the expansion. A number of original...
Taking $T_{44} = \rho c^2$ (all other components zero) in the time component of equation (1) we obtain

$$\left( R_{44} - \frac{1}{2} g_{44} R \right) - \lambda g_{44} = \kappa \rho c^2.$$

This gives on analysis

$$-3\alpha^2/4 + 3\alpha^2/2 - \lambda c^2 = \kappa \rho c^2$$

the second of Einstein’s simultaneous equations.

From the spatial component of equation (1), we obtain

$$\left( R_{ii} - \frac{1}{2} g_{ii} R \right) - \lambda g_{ii} = 0.$$

This gives on analysis

$$3\alpha^2/4 - 3\alpha^2/2 + \lambda c^2 = 0$$

for the first of the simultaneous equations.

It is plausible that Einstein made a sign error here, initially getting

$$3\alpha^2/4 + 3\alpha^2/2 + \lambda c^2 = 0$$

for this equation. (W. Nahm)
Einstein’s steady-state model and cosmology today

- **Accelerated expansion (1998)**
  - Supernova measurements
  - Dark energy – positive cosmological constant

- **Einstein’s dark energy**
  - “The conservation law is preserved in that, by setting the $\lambda$-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”
  - Anticipates positive cosmological constant

- **De Sitter line element**
  - $ds^2 = -e^{at} (dx_1^2 + dx_2^2 + dx_3^2) + c^2 dt^2$ ...
  - Necessary for all steady-state models
  - Identical to inflationary models (different time-frame)
The steady-state universe (1948)

- Expanding but unchanging universe
  *Hoyle, Bondi and Gold (1948)*
  Disliked speculation about physics of early epochs
  Perfect cosmological principle?

- Continuous creation of matter
  Very little matter required
  No beginning, no age paradox

- Replace $\lambda$ with creation term (Hoyle)
  \[ G_{\mu\nu} + C_{\mu\nu} = -k T_{\mu\nu} \]

- Improved version (1962)
  \[ G_{\mu\nu} + \lambda g_{\mu\nu} = k T (C_\mu + C_\nu) \]
Conclusions

**Cosmology – a testing ground for general relativity?**

Assumptions; space-time = space + time
Homogeneity and isotropy
Static universe

**Dynamic cosmology**

Steady-state universe?
Evolving models less contrived

**Evolving models**

Timespan problem: attributed to assumptions
Origins puzzle: ignored

**Verdict**

More data needed
III Astronomy and the Universe

- **The Great Debate (1900-1925)**
  
  *Spiral nebulae = galaxies beyond Milky Way?*

- **The Hooker telescope (1917)**
  
  *Edwin Hubble (1921)*

- **The distances of the nebulae (1925)**
  
  *Cepheid variables resolved in two nebulae*
  
  *Leavitt’s period-luminosity relation*

- **Spirals far beyond Milky Way**
  
  *A universe of galaxies*
The motion of the nebulae

The redshift of the nebulae

V.M Slipher (Lowell Observatory)

Light from most nebulae redshifted (1915, 1917)

Doppler effect

Frequency of light depends on motion of source relative to observer

Nebulae moving outward?

Galaxies moving outward?
Cosmic background radiation

**CMB discovered accidentally**

- Universal signal (1965)
- Low frequency (microwave)
- Low temperature (3K)

*Echo of Big Bang!*

*Penzias and Wilson (1965)*
The big bang model
100 years of relativity

The special theory of relativity (1905)
The general theory of relativity (1916)

General relativity and the universe

The expanding universe (astronomy)
The big bang model

Einstein’s universe

Some new discoveries