One Hundred Years of Cosmology

Einstein, Hawking and the Big Bang

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Midlands Astronomy Club  April 2018
Overview

I  The theory of relativity
   The special theory of relativity
   The general theory of relativity

II  Relativity and the universe
   The static models of Einstein and deSitter
   The dynamic models of Friedman and Lemaitre
   Hubble’s law and the expanding universe

III  The big bang model
   The question of origins
   Hawking-Penrose singularity theorems
   Hawking’s no-boundary universe

Einstein in 1918
Hawking in 2014
Relativity

- The principle of relativity
  Relativity of motion
  Buridan, Oresme, Bruno, Copernicus

- Galileo’s galleon (1632)
  Motion of objects in closed cabin of ship
  Impossible to detect motion of ship by experiments in cabin

- Implications for cosmology
  Motion of earth undetectable to passengers

- Implications for mechanics
  Anticipates Newton’s law of inertia
Relativity in the 19th century

- **Electromagnetism**
  
  *Electricity and magnetism = electromagnetism*
  
  *Speed of electromagnetic wave = speed of light in vac*

- **Light = an electromagnetic wave**
  
  *Changing electric and magnetic fields*
  
  *The electromagnetic spectrum*

- **Speed relative to what?**
  
  *The concept of the ether*

- **The search for the ether**
  
  *Michelson-Morley experiment*
Michelson-Morley experiment

**Expect:** rays should arrive at O out of phase
**Result:** no effect detected
The special theory of relativity (1905)

- **Two principles**
  - Laws of *all* physics identical for observers in relative uniform motion
  - Speed of light in vacuum a universal constant

- **Implications**
  - Intervals in distance and time not universal
  - Experienced differently by bodies in relative uniform motion

- **Predictions (high-speed bodies)**
  - Length contraction: time dilation
  - Mass increase; mass-energy equivalence

- **Minkowski space-time (1908)**
  - Space-time invariant for observers in relative uniform motion
Evidence for SR

- **Mass increase**
  
  *The experiments of Kaufmann and Bucherer*

- **Time dilation**
  
  *The long-lived muon*
  
  $2 \, \mu s \rightarrow 22 \, \mu s$

- **Invariance of the speed of light**
  
  *Always measured as $c$*

- **Particle experiments at the LHC**
  
  *Maximum velocity = $c$*
  
  *Mass increase*
  
  *Particle creation*

\[
m' = \frac{m_0}{\sqrt{1 - v^2/c^2}}
\]

\[
t' = \frac{t_0}{\sqrt{1 - v^2/c^2}}
\]

\[E = mc^2\]
The general theory of relativity (1916)

- **Extending the special theory (1907-)**
  - Relativity and accelerated motion?
  - Relativity and gravity?

- **The principle of equivalence**
  - Equivalence of gravity and acceleration
  - Extension of Galileo’s principle

- **The principle of Mach**
  - Inertial mass defined relative to matter

- **A long road (1907-1915)**
  - Space-time determined by matter
  - Gravity = curvature of space-time
The field equations of GR (1915)

\[ G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \]

10 non-linear differential equations that relate the geometry of space-time to the density and flow of mass-energy

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]

SR  \[ ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \]

\[ ds^2 = \sum_{\mu,\nu=1}^{4} n_{\mu\nu} dx^\mu dx^\nu \]

\[ n_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \]

GR  \[ ds^2 = \sum_{\mu,\nu=1}^{4} g_{\mu\nu} dx^\mu dx^\nu \]

\( g_{\mu\nu} : \) variables determined by matter
Evidence for GR

- **Bending of distant light by stars**
  - Gravitational lensing

- **Gravitational redshift**
  - Shift in wavelength of light due to gravity

- **Gravitational time dilation**
  - GPS corrections

- **Black holes**
  - \[ G_{\mu\nu} = -\kappa T_{\mu\nu} \]
  - Centre of galaxies

- **Gravitational waves**
  - Hulse–Taylor binary system
  - BH, neutron-star mergers
Eclipse Results (1919)

Sobral: 1.98" +/- 0.16  
Principe: 1.7" +/- 0.4

Asymmetric controversy (Collins and Pinch 1970s)
Claim of bias; rebutted by astronomers (RAS)
Relativity and the universe

**Einstein: apply GR to the Universe (1917)**

*Ultimate test for new theory of gravitation*

**Assumptions**

*Uniform, static distribution of matter*

*Mach’s principle: metric tensor to vanish at infinity*

**Boundary problem!**

*Assume cosmos of closed curvature*

*Snag...no consistent solution from GFE*

**New term needed in field equations!**

*Cosmic constant – allowed by theory*

*Anti-gravity effect?*

*Radius and density defined by \( \lambda \)*
De Sitter’s universe

**Alternative cosmic solution for the GFE**
A universe empty of matter (1917)

**Solution B**
Cosmic constant proportional to curvature of space
\[ \lambda = \frac{3}{R} \]

**Disliked by Einstein**
Conflict with Mach’s principle
Problems with singularities? (1918)

**The de Sitter confusion**
Static or non-static - a matter of co-ordinates?
Weyl, Lanczos, Klein, Lemaître

Prediction of redshifts – astronomical interest
The motion of the nebulae

- The redshift of the nebulae
  V.M. Slipher (Lowell Observatory)
  Light from most nebulae redshifted (1915, 1917)

- Doppler effect
  Frequency of light depends on
  motion of source relative to observer

- Nebulae moving outward?
  Galaxies moving outward?
The dynamic universe (theory)

- **Alexander Friedman (1922)**
  - Allow time-varying solutions for the cosmos
  - Two differential equations for R

- **Evolving universe**
  - Time-varying radius and density of matter
  - Considered 'suspicious' by Einstein

- **Georges Lemaître (1927)**
  - Theoretical universe of time-varying radius
  - Expanding universe in agreement with emerging astronomical data
  - Also rejected by Einstein

“Vôtre physique est abominable”
Astronomy and the universe

- **Hubble’s law (1929)**
  
  *A redshift/distance relation for the galaxies*
  
  *Linear relation: $h = 500 \text{ km s}^{-1} \text{ Mpc}^{-1}$*

- **Evidence of cosmic expansion?**
  
  *RAS meeting (1930): Eddington, de Sitter*

- **Friedman-Lemaître models circulated**
  
  *Time-varying radius and density of matter*

- **Einstein apprised**
  
  *Sojourn at Cambridge (June 1930)*
  
  *Sojourn at Caltech (Spring 1931)*
The expanding universe (1930 -)

- **Eddington (1930, 31)**
  *On the instability of the Einstein universe*
  Expansion caused by condensation?

- **Tolman (1930, 31)**
  *On the behaviour of non-static models*
  Expansion caused by annihilation of matter?

- **de Sitter (1930, 31)**
  *Further remarks on the expanding universe*
  Expanding universes of every flavour

- **Einstein (1931, 32)**
  *Friedman-Einstein model*  \( k = 1, \lambda = 0 \)
  *Einstein-de Sitter model*  \( k = 0, \lambda = 0 \)

Expanding models
No mention of origins
Cosmic prediction: the big bang

- **Lemaître 1931**: expanding $U$ smaller in the past
- Extrapolate to very early epochs
- Extremely dense, extremely hot
- Expanding and cooling ever since
- ‘Fireworks beginning’ at $R = 0$?

Not endorsed by community (1930-60)

Singularity problem: timescale problem

Later called ‘The big bang’
A second line of evidence

Gamow and nuclear physics (1940s)
Student of Friedman

How were the chemical elements formed?
Problems with stellar nucleosynthesis

Elements formed in the infant hot universe?
Theory predicts $U = 75\%$ Hydrogen, $25\%$ Helium

Agreement with observation
Support for big bang model?

Heavier atoms formed in stars
Bonus: a third prediction

- Infant universe very hot
- Dominated by radiation
- Radiation still observable today?
  *Low temp, microwave frequency*
- A fossil from the early universe!
  *Released when atoms formed (300,000 yr)*

*Alpher, Gamow and Herman*

No-one looked
The steady-state universe (1948)

- Expanding but unchanging universe
  Hoyle, Bondi and Gold (1948)
  Disliked extrapolation to early epochs
  Perfect cosmological principle?

- Requires continuous creation of matter
  Very little matter required
  No beginning, no age paradox

- Replace $\lambda$ with creation term (Hoyle)
  \[ G_{\mu\nu} + C_{\mu\nu} = -kT_{\mu\nu} \]

- Improved version (1962)
  \[ G_{\mu\nu} + \lambda g_{\mu\nu} = k T (C_\mu + C_\nu) \]
Steady-state vs big bang (1950-70)

- **Optical astronomy (1950s)**
  - Revised distances to the nebulae (Baade, Sandage)
  - New timescale of expansion

- **Radio-astronomy (1960s)**
  - Galaxy distributions at different epochs
  - Cambridge 3C Survey (Ryle)

- **Nucleosynthesis of light elements**
  - Gamow et al. 1948

- **Cosmic microwave background (1965)**
  - Low temperature, low frequency
  - Remnant of young, hot universe
Cosmic background radiation (1965)

**CMB discovered accidentally**

- Ubiquitous signal
- Low frequency (microwave)
- Low temperature (3K)

**Echo of Big Bang!**

Penzias and Wilson (1965)
The big bang – evidence

1. The expansion of the $U$
2. The abundance of $H$ and $He$
3. The distribution of the galaxies
4. The cosmic microwave background

How did it start?
Problems : theory

- The singularity problem revisited
  A facet of simplified assumptions?

- Roger Penrose: black holes
  \( GR \rightarrow BH \) must contain spacetime singularity

- Stephen Hawking: cosmology
  \( GR \rightarrow U \) must begin in a spacetime singularity

- Grand cosmological theorem (Hawking/Penrose)
  “There is a singularity in our past” (PhD, 1966)

**Big bang not a full theory?**
Modern measurements of the CMB

- Details of background radiation
- Full spectrum
- Comparison with theory
- Perturbations?

- Ground telescopes
- Balloon experiments
- Satellite experiments

COBE satellite (1992)
COBE measurements of CMB

- Expected temperature
- Expected frequency
- Perfect blackbody spectrum

- Radiation very uniform
- Variation of 1 in $10^5$
- Seeds of galaxies?

*COBE (1992)*

Nobel Prize
Problems: observation

Background radiation raised new questions

- **Horizon** problem *why so uniform?*
- **Galaxy** problem *how did galaxies form?*
- **Flatness** problem *fine balance?*

Conflict between theory and experiment

Astrophysics: $\Omega = 0.3$ (incl. DM)
Dark Matter

- First suggested in 1930s
- Stellar motion

*normal gravitational effect but cannot be seen directly*

- Explains motion of stars
- Explains motion of galaxies
- Explains gravitational lensing

\[
\text{Matter} = \text{OM (20\%)} + \text{DM (80\%)}
\]

\[\Omega = 0.3?\]
The horizon problem

- Two distant regions of microwave background have similar temps
  
  **Why?**

  *Too far apart to be causally connected*

  - Finite speed of light
  - Finite age of cosmos

**Something wrong with lookback**
Galaxy formation problem

- Microwave background smooth on large scale
- No deviations from homogeneity obvious (1 in 10,000)
- How did slight perturbations become galaxies?
The flatness problem

Slightest deviation from flatness → runaway expansion or crunch

Not observed

Why so finely balanced initially?

\[ \Omega = 1? \]

Astrophysics: \( \Omega = 0.3? \)

At \( t = 1 \text{ s}, \ \Omega = 1 \text{ to within } 1 \times 10^{15} \)
The Theory of Inflation (1981)

- Initial exponential expansion
- Driven by phase transition

**Repulsive force**

- Expansion of $10^{35}$ in $10^{-32}$ s
- Smooths out inhomogeneities
- Smooths out curvature

‘No hair’ universe
The inflationary universe (1981)

- Solves horizon problem
  Early $U$ incredibly small

- Solves flatness problem
  Geometry driven towards flatness

- Mechanism for galaxy formation
  1982 Nuffield workshop (Hawking)
  Quantum fluctuations inflated
  Correct spectrum

$\Omega = 1$?

Conflict between theorists and experimentalists
The accelerating universe (1998)

- Supernovae as standard candles (1998)
- Furthest galaxies too far away
- Expansion accelerating
- Implies repulsive energy
  
  *Dark Energy* $(\lambda \neq 0)$

- Possible support for inflation $(\Omega = 1\,?)$
The flat universe (2000)

- Balloon measurements of CMB
- The BOOMERANG experiment
- The MAXIMA experiment
- Geometry is flat ($\Omega = 1$)
  
  *Implies dark energy component*

  $$\Omega_M + \Omega_\lambda = 1 \quad (\Omega_\lambda = 0.7)$$

- Support for inflation
WMAP Satellite (2002)

- Details of $CMB$ spectrum
- Details of galaxy formation
- Details of flatness of $U$
- Details of dark energy
WMAP measurements of CMB (2005)

- Spectrum of $T$ variations
- Geometry is flat (to 1%)
- Dark energy 74%

**Strong support for dark energy**

**Strong support for inflation**
Modern big bang model: \( \Lambda - CDM \)

A flat, accelerating universe containing matter, dark matter and dark energy

1. Ordinary matter: 4% (astrophysics)
2. Dark matter: 22% (astrophysics)
3. Dark energy: 74% (supernova, CMB)

\( \Omega = 1 \)
Cause of acceleration: dark energy

- Predicted by relativity
- Cosmological constant
- Natural tendency of space to expand

- Energy of vacuum?
- Why so small?
- Why of similar density to matter?

- Not well understood
- Fate of universe?
New results: Planck Satellite (ESA, 2013)

1. Improved sensitivity
\[ \Delta T/T \approx 1 \times 10^{-6} \]

2. Full spectrum of \( T \) anisotropy
   - New acoustic peaks: scale invariance?
   - Accurate values for \( \Omega_\Lambda, \Omega_M \)

3. Gravitational lensing
   - Remove degeneracies

4. Polarization measurements
   - \( E \)-modes: fluctuations
   - \( B \)-modes: gravity waves?
**Planck results (2013)**

1. **New Hubble constant**
   \[ H_0 = 67.3 \pm 1.2 \text{ km/s/MPC} \]
   \[ \text{Age} = 13.8 \text{ billion yr} \]

2. **Curvature; flat**
   \[ \Omega_k = -0.0005 \pm 0.07 \]

3. **New mass/energy parameters**
   \[ \Omega_\Lambda = 68, \quad \Omega_{DM} = 27, \quad \Omega_{OM} = 4.9\% \]
The big bang - problems

- Nature of dark energy?
  *Role in BB?*

- Which model of inflation?
  *The multiverse?*

- The singularity problem
  *What banged?*
  *What does time zero mean?*
  *No-boundary universe*
The no-boundary universe

- Apply quantum physics to spacetime
  
  *Quantum gravity*

- Hawking/Hartle state (1983)
  
  *No spacetime singularity*

- No boundary in time
  
  *Time does not exist before Planck era*

  *Spacetime smeared out*

  “What is north of the North Pole?”
The singularity: a cyclic universe?

- Breakdown at time zero
- No model of bang itself
- Multiple bangs?
- Colliding branes
- Prediction of string theory

- Cyclic universe
- Eternal universe

Tests? Non-Gaussianities in CMB
The big bang model
Where next for general relativity?

- More general theory
  *Unified field theory; the forces of nature (Einstein)*

- Reconcile GR with quantum theory
  *Quantum gravity*

- Some progress
  *Black hole thermodynamics*
  *Hawking-Bekenstein radiation*

- Quantum cosmology
  *The quantum big bang*

*Stephen Hawking*

*A universe from nothing?*
Einstein’s cosmology: conclusions

- Major test for general relativity
  Conscious of assumptions of homogeneity, isotropy

- Embraces dynamic cosmology
  New evidence – new models (JMK)
  Timespan of expanding models puzzling
  Steady-state universe?

- Evolving models (less contrived)
  Cosmic constant not necessary
  Extraction of parameters compatible with observation
  Closed and open models
  Timespan problem attributed to simplifying assumptions

Verdict (1933, 1945): more observational data needed

No mention of origins
Einstein’s lost theory uncovered
Physicist explored the idea of a steady-state Universe in 1931.

Davide Castelvecchi
24 February 2014

New Discovery Reveals Einstein Tried To Devise A Steady State Model Of The Universe

Almost 20 years before the late Fred Hoyle and his colleagues devised the Steady State Theory, Albert Einstein toyed with a similar idea: that the universe was eternal, expanding outward with a consistent input of spontaneously generating matter.

An Irish physicist came across the paper last year and could hardly believe the model of the universe very different to today’s Big Bang Theory. The manuscript, which hadn’t been referred to by scientists for decades, received overwhelming attention as it looks like it could alter our understanding of the universe.

WIT researchers discover ‘lost’ Einstein model of universe
Scientists uncovered misfiled papers while searching Jerusalem university’s online archive.
III  Astronomy and the Universe

- **The Great Debate (1900-1925)**
  
  Spiral nebulae = galaxies beyond Milky Way?

- **The Hooker telescope (1917)**
  
  *Edwin Hubble (1921)*

- **The distances of the nebulae (1925)**
  
  *Cepheid variables resolved in two nebulae*
  
  *Leavitt’s period-luminosity relation*

- **Spirals far beyond Milky Way**
  
  *A universe of galaxies*
A relation between redshift and distance for the galaxies?

Combine 24 distances with redshifts

Redshifts from Slipher: not acknowledged

Linear relation: Hubble’s law (1929)

\[ v = H_0d \] with \( H = 500 \text{ kms}^{-1}\text{Mpc}^{-1} \)

Landmark result in astronomy

Far-away galaxies rushing away

at a speed proportional to distance

Why?
Lemaître’s universe (1927)

- **Expanding model of the cosmos from GR**
  
  Similar to Friedman 1922 model
  
  Starts from static Einstein universe

- **Recession of nebulae = expansion of space?**
  
  Redshifts from Slipher, distances from Hubble
  
  \[ H = 585 \text{ km s}^{-1} \text{ Mpc}^{-1} \]

- **Ignored by community**
  
  Belgian journal (in French)
  
  Rejected by Einstein: “Votre physique est abominable”
  
  Einstein not up-to-date with astronomy?
The expanding universe (1930)

- **RAS meeting (1930)**
  
  *Eddington, de Sitter*

  If redshifts are velocities, and if effect is non-local
  Static cosmic models don’t match observations

- **Expanding universe?**
  
  *Hubble’s law = expansion of space?*
  
  \[ H = 500 \text{ kms}^{-1}\text{Mpc}^{-1} \]

- **Friedman-Lemaître model circulated**
  
  *Time-varying radius*

  *Time-varying density of matter*

  *Evolving universe*
Models of the expanding universe (1930-)

- **Eddington (1930, 31)**
  
  *On the instability of the Einstein universe*
  
  Expansion caused by condensation?

- **Tolman (1930, 31)**
  
  *On the behaviour of non-static models*
  
  Expansion caused by annihilation of matter?

- **de Sitter (1930, 31)**
  
  *Further remarks on the expanding universe*
  
  Expanding universes of every flavour

- **Einstein (1931, 32)**
  
  *Friedman-Einstein model* \( \lambda = 0, \, k = 1 \)
  
  *Einstein-de Sitter model* \( \lambda = 0, \, k = 0 \)

  *Occam’s razor?*
The big bang model

- Afterglow Light Pattern
  - 400,000 yrs.
- Dark Ages
- Development of Galaxies, Planets, etc.
- Inflation
- Dark Energy
  - Accelerated Expansion
- Quantum Fluctuations
- 1st Stars
  - about 400 million yrs.
- Big Bang Expansion
  - 13.7 billion years

WMAP
Einstein’s universe: conclusions

- **Cosmology** = test for general relativity
  *Introduces $\lambda$-term to the field equations*

- **Embraces dynamic cosmology**
  *New evidence – new models*
  *Steady-state vs evolving universe*
  *Evolving models simpler: remove $\lambda$-term*

- **The evolving universe**
  *Extract observational parameters*
  *Timespan problem attributed to simplifying assumptions*

- **No discussion of origins**
  *Wary of extrapolations*

Hubble constant revised

Cosmic microwave background
Homogeneous, flat universe
Einstein’s steady-state model: key quotes

**New solution**

“In what follows, I wish to draw attention to a solution to equation (1) that can account for Hubbel’s facts, and in which the density is constant over time.”

**Matter creation**

“If one considers a physically bounded volume, particles of matter will be continually leaving it. For the density to remain constant, new particles of matter must be continually formed within that volume from space “

**Dark energy**

“The conservation law is preserved in that, by setting the λ-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”
Einstein’s steady-state theory: a significant find?

- **New perspective on steady-state theory (1950s)**
  Logical possibility: not a crank theory

- **Insight into scientific progress**
  Evolution of successful theories
  No Kuhnian paradigm shift to ‘big bang’ model
  Slow dawning

- **Insight into Einstein’s philosophy**
  Simple solution?
  Discards model rather than introduce new term to GFE
  Occam’s razor approach

- **Links with modern cosmology**
  Dark energy, cosmic inflation

*Paradigm shift or slow dawning?*
Explanation for runaway galaxies?

**Newton**

- Gravity pulls in not out
- Space is fixed
- Time has no beginning

*How can galaxies be receding? What is pushing out?*
Results: publications

- **Einstein’s 1931 model**
  

- **Einstein’s steady-state manuscript**
  

- **Einstein-de Sitter model**
  

- **Review paper: conclusions**
Einstein’s steady-state theory: an abandoned model of the cosmos

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Revised 1st February 2014 / Received in final form 12 May 2014
Published online (Accepted Manuscript) 10 May 2014
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Abstract. We present a translation and analysis of an unpublished manuscript by Albert Einstein in which he attempted to construct a steady-state model of the universe. The manuscript, which appears to have been written in early 1931, demonstrates that Einstein once considered a steady-state model in which the mass density of matter in an expanding universe was maintained constant by the continuous formation of matter from empty space. The model was very different to previously
Taking $T_{44} = \rho c^2$ (all other components zero) in the time component of equation (1) we obtain 
\[
\left( R_{44} - \frac{1}{2} g_{44} R \right) - \lambda g_{44} = \kappa \rho c^2.
\]
This gives on analysis 
\[-3\alpha^2/4 + 3\alpha^2/2 + \lambda c^2 = \kappa \rho c^2
\]
the second of Einstein’s simultaneous equations.

From the spatial component of equation (1), we obtain 
\[
\left( R_{ii} - \frac{1}{2} g_{ii} R \right) - \lambda g_{ii} = 0.
\]
This gives on analysis 
\[3\alpha^2/4 - 3\alpha^2/2 + \lambda c^2 = 0
\]
for the first of the simultaneous equations.

It is plausible that Einstein made a sign error here, initially getting 
\[3\alpha^2/4 + 3\alpha^2/2 + \lambda c^2 = 0
\] for this equation. (W. Nahm)
Einstein’s steady-state model and cosmology today

- **Accelerated expansion (1998)**
  - Supernova measurements
  - Dark energy – positive cosmological constant

- **Einstein’s dark energy**
  - “The conservation law is preserved in that, by setting the $\lambda$-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1).”
  - Anticipates positive cosmological constant

- **De Sitter line element**
  - $ds^2 = -e^{\alpha t} (dx_1^2 + dx_2^2 + dx_3^2) + c^2 dt^2$ ...
  - Necessary for all steady-state models
  - Identical to inflationary models (different time-frame)
The most important fact that we draw from experience as to the distribution of matter is that the relative velocities of the stars are very small compared with the velocity of light.

There is a system of reference relative to which matter may be looked upon as being permanently at rest.

In a consistent theory of relativity, there can be no inertia relative to “space”, but only an inertia of masses relative to one another.

I have not succeeding in formulating boundary conditions for spatial infinity. Nevertheless, there is still a way out...for if it were possible to regard the universe as a continuum which is finite (closed) with respect to is spatial dimensions, we should have no need at all of any such boundary conditions.

The most important fact that we draw from experience as to the distribution of matter is that the relative velocities of the stars are very small compared with the velocity of light..... There is a system of reference relative to which matter may be looked upon as being permanently at rest.

However, the system of equations ..allows a readily suggested extension which is compatible with the relativity postulate... For on the left hand side of the field equation...we may add the fundamental tensor g_{\mu\nu} , multiplied by a universal constant , — \lambda, at present unknown, without destroying the general covariance.

Some key quotes (Einstein 1917)

Schroedinger’s comment (1918): Einstein’s response (1918)
An abandoned model

- **Correct geometry**
  - de Sitter metric

- **Simultaneous equations**
  - Eliminate $\lambda$
  - Relation between $\alpha^2$ and $\rho$

- **Einstein’s crossroads**
  - Null solution on revision
  - Tolman? (Nussbaumer 2014)
  - Declined to amend GFE

- **Evolving models**
  - Less contrived: set $\lambda = 0$
Steady-state universe (1948)

- Alternative to big bang *(Fred Hoyle)*
- Expanding universe

**BUT**

- Continuous creation of matter?
- Unchanging universe
- No beginning, no age problem
- No assumptions about early epochs

Very little matter needed
3. Einstein’s steady-state model

Unpublished manuscript
Archived as draft of F-E model (1931)
Similar title, opening to F-E model

Something different
Cosmological constant
“The density is thus constant and determines the expansion”

Steady-state model of the Expanding Universe
Anticipates Hoyle solution
Written in early 1931
Fatal flaw: abandoned

\[
9\alpha^2 / 4 + \lambda c^2 = 0
\]
\[
3\alpha^2 / 4 - \lambda c^2 = \kappa \rho c^2
\]
\[
\alpha^2 = \frac{\kappa c^2}{3} \rho
\]
Einstein’s lost theory uncovered

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2. Einstein-de Sitter model (1932)

- **Remove spatial curvature**
  
  Curvature not a given in dynamic models (Heckmann)
  
  Not observed empirically (Occam’s razor)

  \[ ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2dt^2 \]

- **Simplest Friedman model**
  
  Time-varying universe with \( \lambda = 0, k = 0, p =0 \)
  
  Estimate of density: \( \rho = 10^{-28} \text{ g/cm}^3 \)

- **Becomes standard model**
  
  Despite high density of matter, age problem
  
  Time evolution not considered

- **Longer version with time evolution (Einstein 1933)**
IV The ‘big bang’ model (1931)

- Infant $U$ concentrated in tiny volume
- Extremely dense, hot
- Expanding and cooling ever since

*Where do the laws of physics come from?*

*Wrong age (Hubble constant)*

Singularity problem

$\infty$ density, $\infty$ temp at $t = 0$?
Cosmic prediction I: Black Holes

**Schwarzschild (1916)**

*Exact solution for the field equations*

*Body of spherical symmetry*

**Enigma**

*Solution becomes singular at* $r = \frac{2GM}{c^2}$

*Space closed up around mass?*

**Rejected**

*Co-ordinate problem (Eddington)*

*Prevented by internal pressure (Einstein 1922)*

**Physical reality?**

*Collapse of sun? Anderson (UCG)*

*Collapse of large stellar ensemble: Lodge (Oxford)*
The physics of black holes

- **Chandrasekhar (1931)**
  - *The physics of white dwarf stars (quantum degeneracy)*
  - SR: collapse to infinite density for $M > 1.4 \, M_\odot$
  - Rejected by Eddington, community

- **Oppenheimer (1939,40)**
  - GR: Continued stellar collapse for $M > 3 \, M_\odot$
  - Rejected by Einstein (1939)

- **Wheeler, Thorne, Zeldovitch (1960s)**
  - Numerical solutions of the field equations
  - Simulation of stellar collapse

- **Penrose (1965)**
  - No avoiding BH singularity
Black Holes: Observation

**Compact astronomical objects (1960s)**

*Quasars: small, distant sources of incredible energy (1963)*
*Pulsars: rapidly rotating neutron stars (1967)*

**X-ray binaries**

*Cygnus X-1 (1964)*

*Matter pulled from star into massive companion emits X-rays*

**Orbit studies**

*Supermassive BH at centre of MW? (1990s)*
*Supermassive BH at centre of many galaxies (2000-)*

**2015-16**

*Gravitational waves from binary BH system!*

*Quasar 3C273*

*Cygnus X-1 (1964)*
Relativity and the universe

The field equations of general relativity (1916)

\[ G_{\mu \nu} = -\kappa T_{\mu \nu} \]

Solution for the case of the universe?

Ultimate test for new theory of gravitation

Assumptions

Uniform, static distribution of matter
Closed spatial curvature
Introduce the cosmic constant \( \lambda \)

The Einstein World (1917)

Static universe of spherical geometry
Cosmic radius and matter density defined by \( \lambda \)

\[ \lambda = \frac{\kappa \rho}{2} = \frac{1}{R^2} \]
Big bang puzzles

- **Characteristics of background radiation**
  Homogeneity, flatness, galaxy formation? (1970-80)

- **The theory of inflation (1981)**
  Exponential expansion within first second?
  Initial conditions?
  Which model of inflation?

- **Dark energy (1998)**
  Observation of accelerated expansion
  The return of the cosmological constant
  Problems of interpretation

\[ G_{\mu\nu} + \lambda g_{\mu\nu} = -\kappa T_{\mu\nu} \]

*Nature of DE unknown*
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- **Orbit studies**
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  - Supermassive BH at centre of many galaxies (2000-)

- **2015-16**
  - Gravitational waves from BH merger!
Three astronomical tests (Einstein, 1916)

- **Different in principle from Newton’s gravity**
  *Small deviations in practice (weak scale)*

- **The perihelion of Mercury**
  *Well-known anomaly in Mercury’s orbit (43" per century)*
  *Postdicted by GR (1916)*

- **The bending of starlight by the sun (1.7")**
  *Eclipse expeditions of Eddington and Dyson (1919)*
  *Successful measurement (large error margin)*

- **Gravitational redshift**
  *Time dilation in strong gravitational field*
  *Light from a star redshifted by stellar mass?*

- Einstein-de Sitter: $\lambda = 0, k = 0$

- Flatness prediction (Dicke, inflation)
  \[ k = 0 \]

- But: $\Omega_M = 0.3$

- But: new timescale problem from HST
  \[ H = 87 \text{ kms}^{-1} \text{ Mpc}^{-1} : t = 11.2 \times 10^9 \text{ yr} \]
  too young

Is model wrong? Is $\lambda \neq 0$?